A Theory of Party Evolution

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Abstract

Multiparty systems often experience significant changes due to parties' splits and mergers. While sometimes splitting provides a clear electoral benefit, other times factions split without an immediate return. We present a dynamic model to analyze the evolution of parties, accounting for both scenarios. In our model, factions decide over time whether to split or stay together based on incentives to cultivate their political power. We characterize when parties remain united, fragment, and exhibit reversals of splits and re-mergers. Our analysis distinguishes between consensual and unilateral splits: some occur cooperatively, as factions temporarily separate to strengthen their joint position before reuniting, while others are conflictual, driven by one faction's attempt to improve its relative standing upon re-merger. Both types may arise even when splitting is electorally damaging. Finally, we show that institutional features often thought to preserve unity—such as disproportional electoral systems or egalitarian internal rules—may, under certain conditions, instead encourage fragmentation.

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1. Introduction

Political parties are rarely monolithic actors. They comprise factions—organized around ideological currents, regional constituencies, or competing leaders—that often rearrange themselves through splits and mergers. Recent cases abound: In Germany, Sahra Wagenknecht's break from *Die Linke* in 2023 to form a new party immediately altered the balance on the left; in Japan, the 2017 split of the Democratic Party produced the Constitutional Democratic Party, reshaping the balance of the opposition. In Brazil, political survival pressures led the Brazilian Labour Party and Patriota to merge in 2023, forming the Democratic Renewal Party (PRD). Such episodes show how realignments can reshape the trajectory of individual parties, a pattern echoed by systematic evidence of more than two hundred post-war schisms and frequent mergers in Europe (Ibenskas, 2019), as well as studies documenting instability in Latin America and Eastern Europe (Mainwaring, 1999; Tavits, 2008). Figure 1, reproduced from Chiru et al. (2020), illustrates the frequency of such events across 35 democracies around the world, showing that both splintering and merger episodes are far from exceptional.

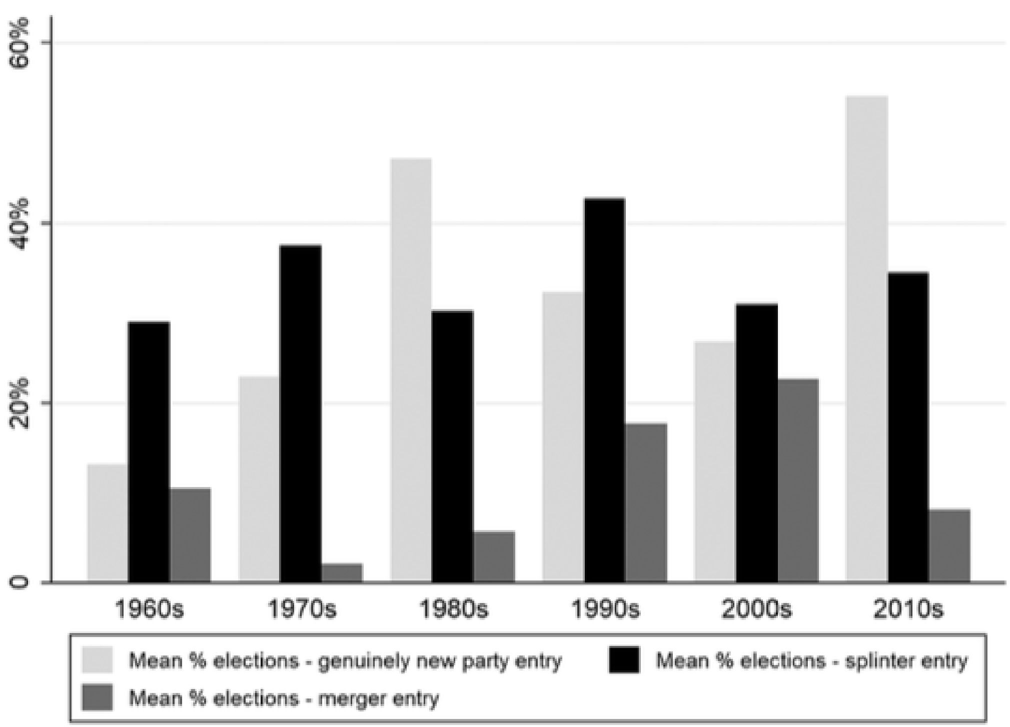

Figure 1 – Frequency of splinter and merger parties, 1945–2015. Source: Chiru et al. (2020).

Sometimes, these realignments follow an intuitive logic. A faction breaks away because it expects to be more successful on its own, attracting new supporters and redefining the ideological landscape. This was the case of the UK Social Democratic Party, formed in the early 1980s by a group of Labour moderates who sought to appeal to centrist voters. The splinter, now the Liberal Democratic Party, succeeded in reshaping British party competition, and the fragmentation it triggered proved stable. The Liberal Democratic Party is, to this day, a rare example of a stable third party in a first-past-the post electoral system.

Other times, however, factional departures are more puzzling. A group may initiate a split despite anticipating little independent success or even a loss of influence. In 2017, for instance, Pierluigi Bersani and other left-wing politicians left Italy's *Partito Democratico* to form a new party, *Articolo 1*. The move weakened the center-left camp overall and failed to generate a viable alternative. One may be tempted to interpret these developments as the consequence of strategic mistakes or miscalculations by the faction's leaders. Interestingly, however, *Articolo 1*'s poor electoral performance was widely anticipated.[1] Indeed, Bersani openly acknowledged the cost: "It's legitimate to think that we are barking at the moon, there are no tangible results yet. We're doing this more for future memory than for the concrete present."[2]

This paper develops a unified framework to analyze these dynamics. Our approach emphasizes factions' strategic incentives to cultivate electoral support and to anticipate how today's organizational choices affect tomorrow's political power. Our model not only accounts for intuitive outcomes, such as parties remaining united to benefit from running together, or splits initiated by a faction that expects to gain more power on its own, but also rationalizes puzzling dynamics—most notably damaging splits, in which parties break apart despite anticipating short-term losses, only to re-merge later. These equilibria, while counterintuitive in a static setting, capture important episodes in the evolution of real-world parties which we further discuss in the paper.

Our theory builds on two key ingredients. First, factions within the same party possess distinct identities and, as a result, different bases of electoral support (Clarke, 2020). Voters

[1] In a public interview, Bersani explained the necessity of establishing a separate political entity, despite the grim electoral prospects: https://www.ilfattoquotidiano.it/2017/02/28.

[2] Party national assembly, November 16, 2019 (minute 9): https://www.youtube.com/watch?v=T5rrWSuyH6M.

evaluate each faction's credibility, the competence of its members, and the appeal of its ideological stances. These features produce distinct constituencies that underpin the party's success and determine each faction's influence in internal negotiations, as well as its outside option if it runs alone. Even under a common party label, such independent support is central to each faction's strategic positioning. As Clarke (2020, p. 456) puts it, "party sub-branding is thus a crucial element in the factional politics of resource capture." Our model takes this as its starting point: factions leverage their independent support to consolidate power both within and outside the party.

Second, a split from the main party can substantially reshape each faction's standing among the electorate. A faction may be more appealing to ideologically aligned voters when it runs independently, or benefit from an increased visibility or clearer messaging (Lo, Proksch and Slapin, 2016). Conversely, voters may be skeptical of the motives underlying the split or find the faction's new independent identity less appealing. A split thus alters the configuration of electoral support, changing the balance of power both within and across ideological camps. These realignments generate the dynamic strategic logic driving our results.

In the model, factions' payoffs depend on their electoral support and on the organizational form they choose. When factions remain together, they share the collective benefits of unity—the disproportionality in most electoral systems generally favors larger parties, creating an efficiency premium to running together. At the same time, a faction's share of these benefits reflects its relative support among voters. When a faction runs independently, its electoral support directly translates into political power, but it forgoes the efficiency premium of unity. Crucially, as mentioned above, the act of splitting itself reshapes electoral support: a breakaway can enhance a faction's visibility and attract new voters, or it can undermine credibility and alienate existing constituencies. Finally, broader ideological trends—such as shifts in the electorate's position or the strength of the party's ideological camp—affect all factions' prospects, creating periods when the future looks more favorable than the present.

The model generates both intuitive and surprising results. We first analyze a static benchmark of the game. In a one-shot setting, whether factions remain united or split depends entirely on the balance between the efficiency gains of staying together and the electoral consequences of breaking apart. When unity yields a clear premium—through a strong effect of disproportionality or because a split would damage both factions' electoral appeal—factions can always bargain over how to share these benefits, making separation unattractive. By contrast, when splitting significantly expands the camp's electoral reach, perhaps because both factions can more effectively mobilize their natural bases of support, the added support outweighs the value of unity, and no internal deal can hold the party together. Put simply, parties fragment in static terms only when division creates a surplus of resources, compared to the factions staying together. Otherwise, when a split destroys surplus, the incentives always favor unity.

The dynamic logic of the model, however, qualifies this intuition. In the repeated game, splits may occur even when they destroy surplus in the short run. A statically optimal split still arises in equilibrium (since it never reduces the party's future prospects), but dynamic incentives also open the door to splits that are not statically optimal. One possibility is that a splinter faction breaks away today in order to increase its relative size and bargaining position tomorrow, anticipating a re-merger under more favorable terms. Another is that a split modestly expands the camp's support—though not enough to be statically optimal—and factions nonetheless separate in the first period in order to reunite later as a stronger party. In both cases, temporary fragmentation becomes a rational strategy, even though it would never emerge in a static setting.

Most strikingly, the model rationalizes the occurrence of splits that are damaging to the whole camp. Our analysis shows that these equilibria can arise because splitting today reshapes bargaining power tomorrow, making temporary fragmentation a rational strategy. This logic operates through what we call the "bigger fish in a smaller pond" dynamic, where one faction initiates a split that damages the camp as whole in order to improve its own relative standing within the party tomorrow, thus getting a bigger share of a smaller pie. Here, both factions can be hurt by a split, but the splinter suffers relatively less. By weakening its opponent more than itself, the splinter faction increases its relative weight within the party upon re-merging, thereby securing a stronger bargaining position in the future. This result helps explain the split of Articolo 1 from Italy's Democratic Party (PD). Consistent with our theory, the split was understood as both costly (for the splinters *and* the left-wing camp as a whole)[3] and temporary.

[3]In the last election before the split (2013), the PD (in coalition with other left-wing parties) obtained roughly 29% of the votes. Immediately after the split in 2018, the PD (plus its coalition) and Articolo 1 (plus its coalition)

Bersani himself repeatedly indicated that Articolo 1 would consider rejoining the PD under the right conditions. Such conditions materialized in 2023, when Articolo 1 merged back into a weakened PD from a position of increased relative strength.

Because these dynamic gains come at the expense of the other faction, splits sustained by the "bigger fish in a smaller pond" logic are never consensual: they can only be initiated unilaterally. This aligns with evidence from the case described above: Articolo 1 splintered despite explicit attempts at appeasement by the other faction. The analysis also shows that such behavior requires sufficiently favorable future prospects for the camp as a whole. In the model, this corresponds to an upward ideological shift in the camp's electorate, which enlarges the total pool of potential supporters for both factions. Absent the expectation of a more favorable electorate in the future, the factions could always strike a Pareto-superior deal to remain united. This logic highlights a form of dynamic resource curse: when parties anticipate abundant resources tomorrow, factions may still choose to fracture today, even when the split is dynamically inefficient and the other side is willing to concede the entire pie to preserve unity. This dynamic also echoes empirical patterns whereby party splits often emerge not during decline, but at moments of rising popularity, when future prospects are especially favorable (Lupu, 2016; Ibenskas, 2019).

We next analyze cases where, although a split increases the camp's support, the effect is too small to offset the efficiency premium from unity (and thus the split remains statically inefficient). The trade-off then differs from what described above. One option is for the factions to choose unity today to enjoy the efficiency premium, and either split tomorrow (to enjoy the support-enhancing effect of a split, but giving up the efficiency premium) or keep the party united once again (to enjoy the efficiency premium but giving up the effect of a split). Alternatively, the factions can split today—incurring the immediate loss of the premium while raising total support—and re-merge tomorrow to enjoy both the efficiency premium and the higher support. Which option prevails again depends on the anticipated ideological trend in the electorate. Because the efficiency premium amplifies favorable trends when factions are united, a statically inefficient split arises if and only if the trend is not too negative: the electorate's ideological

obtained a combined vote share of roughly 26%. We can't attribute this difference to the split directly, but the evidence is at least consistent with our expectations.

drift is not expected to be too unfavorable, so the gains from tomorrow's re-merger exceed the immediate cost of splitting.

In sharp contrast to splits that alienate supporters from the ideological camp, a support-building split can be consensual: both factions prefer to separate today while anticipating a future reunion. In such cases, one faction is willing to forgo the immediate efficiency gains from unity and accept reduced bargaining power within the party tomorrow. The logic follows that of a "smaller fish in a bigger pond": the seemingly weaker faction accepts a smaller relative position to allow the party to expand, ultimately securing a smaller share of a larger pie. Consistent with this logic, such episodes should display little open conflict and no efforts to prevent separation, as the split is dynamically efficient for both factions. This result provides a potential rationale for the split of *The People's Party* from the Social Democratic Party in Iceland in the mid-1990s.[4] As we further discuss below, the split was explicitly consensual and aimed at increasing the social-democratic camp's reach in the electorate.[5] The two parties maintained cooperative relations, even running joint lists in several local coalitions, and ultimately re-merged in 2000 in a stronger united party that built on the increased support of its predecessors.

We next turn to the role of electoral institutions in shaping party evolution. The classic Duvergerian intuition holds that disproportionality in the electoral system should discourage splits, since larger parties benefit from an efficiency premium—the bonus of running together rather than separately (Duverger, 1951). In a static framework, this logic is straightforward: as the rewards from unity increase, factions can always find a bargaining arrangement that preserves cohesion. In our dynamic setting, however, this very efficiency premium can become the source of instability. Precisely because unity promises higher future returns, it also creates incentives for factions to engineer temporary separations that reposition them to capture a larger share of those gains.

[4]A concise account of the People's Party's formation and trajectory, published on the party's official website: https://xs.is/sogulegt-agrip-flokksins.

[5]While it is hard to isolate the effect of the split, evidence suggests it did help increase the electoral reach of the social-democratic camp: in the last elections before the split (1991), the Social Democratic Party obtained 15.5% of the votes. In the first election immediately after the split (1995), the combined vote-shares of the two parties increased to 18.6%.

Finally, we build on these results to analyze the effect of party internal institutions on party evolution. A central question is how the rules that govern power sharing within parties shape factions' incentives to split or remain united. In our framework, a party's internal organization determines how sensitive the distribution of resources and influence is to differences in factional support. In more egalitarian arrangements, even relatively small factions retain a meaningful share of power, while in less egalitarian ones, small differences in support translate into large disparities in control. We show that the impact of such rules on stability crucially depends on the effect of a split on the camp as a whole: when splits are damaging, egalitarian institutions help deter fragmentation, but when splits are beneficial the opposite is sometimes true. As detailed below, our results on the effect of electoral and intra-party institutions resonate with the mixed empirical patterns observed across party systems.

We conclude this section by emphasizing that our proposed mechanism is one among many forces shaping party evolution. Existing research has underscored the importance of institutional arrangements such as electoral systems (Golder, 2006*b*,*c*; Blais and Indridason, 2007) and of shifts in voter preferences that increase fragmentation in the electorate (Rokkan and Lipset, 1967; Pedersen, 1979; Taagepera and Grofman, 2003; Invernizzi, 2023). Our framework incorporates these macro-level factors but shifts the focus to the micro-foundations of party change: the strategic interaction among factions operating within existing institutional and electoral environments. By highlighting how forward-looking factions anticipate the future payoffs of unity or separation, our theory complements institutional and voter-based explanations and helps reinterpret episodes often dismissed as miscalculations or leadership disputes as rational responses to dynamic incentives.

2. Related Literature

Our theory is based on the premise that parties are internally divided into competing factions. The formal literature has increasingly acknowledged the importance of factions to understand political parties' nomination processes (Caillaud and Tirole, 2002; Crutzen, Castanheira and Sahuguet, 2010; Hirano, Snyder Jr and Ting, 2009), intra-party power sharing (Invernizzi, 2022; Invernizzi and Prato, 2024), and competition, both over resources (Persico, Pueblita and Silverman, 2011) and ideology (Izzo, 2023). We share with this literature the focus on within-party actors, political factions. We show how considering factional incentives to increase their power leads to unexpected predictions on party evolution.

The literature on American and comparative politics has put forward a few alternative hypotheses for why parties emerge and change. One approach focuses on the demand side, highlighting voters' heterogeneous preferences as the key explanation for party emergence. According to this primordialist account (Rokkan and Lipset, 1967), parties originate as a consequence of social cleavages, and the more numerous the cleavages, the higher the number of parties. An opposite "top-down" approach is the one taken by Downs (1957) and subsequently revisited by Aldrich (1995), according to whom parties are set in motion by career-concerned politicians who need an institutional machinery to support them in elections and once in office. In this tradition, Snyder and Ting (2002) study how the party leadership uses control of the party platform to more effectively signal the candidates' preferences to voters. Levy (2004) analyzes party formation in the presence of a multidimensional policy space, where policy-motivated politicians can form coalitions (parties) to credibly commit to a broader set of policies (the Pareto set of the coalition). We also model party formation, and dissolution, as a top-down process, but instead of focusing on elite coordination over platforms, we highlight factions' dynamic incentives to form new political entities through party exit.

Related models have typically focused on *party entry* as a determinant of party system evolution. For instance, Buisseret and Van Weelden (2020) study how an outsider candidate decides to enter the electoral contest (either via primaries or via a third-party), while Kselman, Powell and Tucker (2016) focus on party entry in Proportional Representation systems. Closest to our model, Forand and Maheshri (2015) consider how party systems evolve in a dynamic setting under different electoral systems. In their model, dynamic considerations arise due to exogenous stochastic changes to voters' ideological preferences and frictions in the electoral process for newly formed parties (i.e., an electoral penalty and higher resource demands). We complement this paper by focusing on factions' dynamic incentives to cultivate their influence. This mechanism generates new results, such as equilibrium reversals of splits and re-mergers, including damaging splits that factions undertake despite short-term costs.

In our model, factions that remain within the same party bargain over the division of the resources they expect the party to gain in the upcoming election. Our bargaining protocol follows classic model of legislative bargaining such as Baron and Ferejohn (1989) with random recognition rule. In our setting, the probability of recognition is tied to a faction's relative electoral strength, capturing the intuition—supported by empirical evidence—that political power and resources allocated to each faction are a function of its electoral support (Diermeier, Eraslan and Merlo, 2003; Warwick and Druckman, 2006; Golder, 2006*a*). Our "dynamic resource-curse" result—that the expectation of abundant future resources can make separations unavoidable—parallels Powell (2006)'s account of war as a commitment problem, where exogenous power shifts can undermine credible agreements even under complete information. In our model, by contrast, changes in relative power emerge endogenously: we show that the very anticipation of greater resources can induce factions to engineer such power shifts through statically inefficient splits. This logic of bargaining failure complements other sources of inefficiency that we do not model. For example, asymmetric information about the electoral or organizational consequences of a split could also produce bargaining breakdowns, echoing the standard incentive-to-misrepresent mechanism in the conflict literature (Fearon, 1995*a*).

Finally, our paper connects to research on party switching, where candidates or legislators change party affiliation. This literature focuses on the incentives of individual politicians, emphasizing the immediate electoral, office, and policy benefits and costs associated with party switching (e.g., Desposato, 2006; Mershon and Shvetsova, 2013*a*,*b*). Our theory differs in two respects. First, we highlight the importance of considering actors' dynamic incentives, showing that we may observe party splintering even when it is costly in the short-run. Second, we think about coordinated groups of party members (i.e., factions), rather than individual politicians, as the key actors. As such, our model can inform recent empirical work that analyzes collective switches from legislatures into new parliamentary groups.[6]

[6]See the Party Instability in Parliaments (INSTAPARTY) Project: https://instapartyproject.com.

3. A Model of Party Evolution

Consider a game between two leftist factions, A and B, a (non-strategic) unified right-wing party, R, and a unit mass of voters. At the beginning of the game, the two factions belong to the same party L. If the factions remain together in the first period, they agree—according to a bargaining protocol described below—on how to divide the resources they expect to obtain from the upcoming election. For example, parties determine the composition of the electoral lists (assigning different candidates to safe or contested spots), or bargain over the allocation of portfolios or patronage opportunities. Otherwise, each faction can unilaterally choose to split and form its own party.

Each voter decides whether to vote and, if so, which party to support based on ideology, and on each party/faction's valence, which captures its overall electoral appeal or "brand strength". Crucially, a split within the left-wing camp affects the factions' valences. By separating, a faction may gain visibility and clarify its message, thereby increasing its electoral appeal; alternatively, it may harm its image if voters view the new independent identity as unappealing or interpret the split as divisive or opportunistic.

The electoral results determine the allocation of power and resources across parties. The game then proceeds to a second period, in which the factions may choose to reunite if they both wish to do so before another round of internal bargaining and a second election take place.

Voters

Each voter i derives a participation utility term $c \in \mathbb{R}$, which captures the psychological or expressive value of voting. A positive c represents a "warm-glow" benefit from participation, while a negative value captures the disutility of time, effort, or inconvenience associated with casting a ballot. Net of this term, voting for party or faction j yields utility $v^j_{\theta_i}$.

Voters differ by ideological type $\theta_i \in \{\ell, r\}$. In period t, a share λ_t of voters are left-wing and the remainder $1 - \lambda_t$ are right-wing. To capture ideological trends, λ_t evolves over time, with $\lambda_t \in [\underline{\lambda}, \overline{\lambda}]$ and $0 < \underline{\lambda} < \overline{\lambda} < 1$.

Right-wing voters associate negative valence with left-wing factions. In particular, we assume that $v^A_r = v^B_r = -\infty$. Thus, a right-wing voter may only support the right-wing party R, with valence $v_r^R \sim g_R$, where g_R represents a continuous distribution with support on $\mathbb{R}$, mean $\bar{v}^R$ and CDF G_R. She votes for R if $v_r^R + c > 0$ and abstains otherwise.

Symmetrically to what defined above, for a left-wing voter $v_\ell^R = -\infty$. Thus, a left-wing voter may only support factions within the left camp. Each faction $j \in \{A, B\}$ has valence $v_\ell^j \sim g_j$ with mean $\bar{v}^j$ and CDF G_j.[7] A split between factions affects their electoral appeal by shifting expected valence: if a split occurs, the mean shifts from $\bar{v}^j$ to $\bar{v}^j + \delta_j$, where $\delta_j \in \mathbb{R}$.[8] A positive δ_j reflects a visibility or clarity gain from running independently, while $\delta_j < 0$ captures voter aversion to internal division or to the faction's new image. The effect of a split on expected valence persists over time.

Upon observing the value of v_ℓ^A and v_ℓ^B, a left-wing voter chooses whether and how to vote. If the factions remain together as a single party, a left-wing voter votes for the party if and only if $\max\{v_\ell^A, v_\ell^B\} > c$, and abstains otherwise. This formulation captures that even within a unified party, factions retain distinct identities (e.g., via primaries or caucuses), but the ballot is cast for the common party label. If the factions run separately, each left-wing voter chooses whether to abstain or vote for one of the two parties: A if $v_\ell^A > \max\{v_\ell^B, c\}$, B if $v_\ell^B > \max\{v_\ell^A, c\}$, and abstains otherwise.

We note that this formulation, where a left-wing voter would never support a right-wing party and vice versa, allows us to separate the exogenous effect of ideological trends from the endogenous effect of a split, ensuring that changes in the factions' relative support across periods reflect strategic rather than mechanical shifts.

The Political System

The parties' vote shares determine the allocation of political power and resources following the elections. Denote Σ_t the history of splits in the left-wing camp at time t. Specifically, $\Sigma_t = 0$ if no split has occurred on the equilibrium path up to and including period t, and $\Sigma_t = 1$ if a split has occurred in period t or earlier. In other words, Σ_t indicates whether the factions have ever split before time t. In what follows, $S_t^j(\lambda_t, \Sigma_t)$ denotes j's vote-share at time t, which in equilibrium will be a function of λ_t and the history of splits Σ_t.

[7] We make no further assumptions on g_j beyond continuity and full support on $\mathbb{R}$; the results are invariant to their specific form.

[8] The shift affects only the mean of the valence distribution; its shape remains unchanged.

If the factions split and contest elections as distinct parties, faction j's political power in period t is

$$f\left(S^j(\lambda_t, 1)\right). \tag{1}$$

If instead the factions are united, the party's power is

$$f\left(S^A(\lambda_t, \Sigma_t) + S^B(\lambda_t, \Sigma_t)\right). \tag{2}$$

Notice that, even if the parties are united at time t, Σ_t may take value 1, reflecting a split occurred in the past. This is why the vote-shares under unity are written for a generic Σ_t, while the vote-share under a split is always evaluated at $\Sigma_t = 1$.

The $f(\cdot)$ function thus captures political power or success: the size of the political "pie" won by the faction/party. We assume f is continuously differentiable, $f' > 0$ and $f'' \geq 0$. The fact that $f(\cdot)$ is (strictly) increasing in its argument reflects the idea that greater popular support typically translates into greater influence over political outcomes.[9] Furthermore, we assume the $f(\cdot)$ function is weakly convex, which implies that — *everything else being equal* (i.e., net of the effect of the split on the factions' support) — the factions' total political power is higher if factions run together than if they run separately. Below, we discuss interpretations and scope conditions for this assumption.

We impose the following assumptions:

Assumption 1. *If* $S_t^A(\lambda_t, \Sigma') + S_t^B(\lambda_t, \Sigma') > S_t^A(\lambda_t, |1 - \Sigma'|) + S_t^B(\lambda_t, |1 - \Sigma'|)$, *then*

$$f\Big(S_t^A(\overline{\lambda}, \Sigma') + S_t^B(\overline{\lambda}, \Sigma')\Big) - f\Big(S_t^A(\overline{\lambda}, |1 - \Sigma'|) + S_t^B(\overline{\lambda}, |1 - \Sigma'|)\Big) \quad \textit{is sufficiently large.}$$

and

Assumption 2. *If* $S_t^A(\lambda_t, \Sigma') + S_t^B(\lambda_t, \Sigma') > S_t^A(\lambda_t, |1 - \Sigma'|) + S_t^B(\lambda_t, |1 - \Sigma'|)$, *then*

$$f\Big(S_t^A(\underline{\lambda}, \Sigma') + S_t^B(\underline{\lambda}, \Sigma')\Big) \quad \textit{is sufficiently small,}$$

[9] Small parties can sometimes exert disproportionate power—especially in bargaining environments like coalition formation—but these are context-specific exceptions. Our assumption abstracts from such nuances to capture the broader relationship between support and political strength.

and

$$f\Big(S_t^A(\overline{\lambda}, |1-\Sigma'|) + S_t^B(\overline{\lambda}, |1-\Sigma'|)\Big) \quad \textit{is sufficiently large.}$$

Recall that $\Sigma \in \{0,1\}$, therefore Σ' as defined above is the most favorable split history: the one that, net of the ideological trend, maximizes the camp's support. Thus, taken together these assumptions guarantee that, for all parameter values, both the exogenous effect of the ideological trend and the endogenous effect of a split on the party's success remain meaningful. For example, the function $f(x) = \frac{1}{1-x}$ satisfies both these assumptions, with $\underline{\lambda} \to 0$ and $\overline{\lambda} \to 1$.

Factions' Payoffs and the Bargaining Protocol

Factions care about political power. When faction j splits and runs alone, its period-t payoff is

$$f\left(S^j(\lambda_t, 1)\right). \tag{3}$$

When the factions run together, both contribute to electoral performance but must share resources. Faction j's period-t payoff is

$$x_t^j\, f\left(S^A(\lambda_t, \Sigma_t) + S^B(\lambda_t, \Sigma_t)\right), \tag{4}$$

where $x_t^j \in [0,1]$ is the share of party resources allocated to faction j at time t. This share is an equilibrium object determined by the following bargaining protocol.

Let faction j's relative size in period t be

$$\rho_t^j(\lambda_t, \Sigma_t) \equiv \frac{S_t^j(\lambda_t, \Sigma_t)}{S_t^A(\lambda_t, \Sigma_t) + S_t^B(\lambda_t, \Sigma_t)}. \tag{5}$$

Faction j is recognized as the proposer in period t with probability $\pi^j(\rho_t^j)$, where $\pi^j : [0,1] \to [0,1]$ is increasing. This can be interpreted, for example, as the probability that faction j controls the party leadership (e.g., via a primary). The recognized proposer then offers a resource allocation $(x_t^j, 1 - x_t^j)$. Each faction subsequently decides whether to accept and remain within the party or to split and form a new party.

Timing:

1. One faction is recognized as a proposer

2. The proposer chooses whether to leave the party or propose an allocation

3. The other faction chooses whether to accept the allocation or leave the party

4. The first-period election is held, each voter chooses if and how to vote, and the first-period payoffs are realized

5. If the factions begin period 2 in the same party, the game proceeds as above. If the factions begin period 2 apart, they can re-merge if they both accept to do so. The game then proceeds as above.[10]

3.1. Discussion of the Assumptions

Before concluding this section, we briefly discuss some of our modeling choices and assumptions.

First, we model political power as a convex function of electoral support. While a linear f captures a perfectly proportional electoral system in which each additional vote translates into the same increase in power, we capture disproportional systems by considering a convex f. In practice, disproportionality generates distortions that advantage relatively larger parties, due to features such as legal thresholds (e.g., minimum vote shares to obtain seats), seat bonuses/premia for the leading party, and winner-take-all components (e.g., plurality districts), each of which magnifies gains for larger parties relative to smaller ones. A convex function mapping support to power reflects these features. However, an important caveat is in order. In reality, the distortions induced by disproportionality are small for very large parties: At very high vote shares, the marginal seat gain often flattens due to finite house size, upper apportionment caps, and diminishing returns in plurality districts already won, yielding an S-shaped mapping from votes to power that is convex at low–mid shares and concave at high shares. By focusing on the range where f is convex (before it flattens), we limit the scope of our theory to parties that are not so dominant that they would secure the entire pie even under adverse ideological trends or following a negative split. Similarly, we exclude factions that would secure the entire pie even without unifying (in which case sustained unity is trivially impossible).

[10]We could allow the proposal stage before the re-merging decision. It would not change much.

Second, we assume that bargaining among factions happens before the election, and that factions commit to the division of party resources they would obtain after the election. Alternatively, one can imagine that factions could bargain only after resources are realized—i.e., after the election—and at that point choose whether to accept the proposed division or exit the party. We could enrich the model to incorporate this possibility, but it would require adding a third period (a third election). If a split occurs at the end of the first period, its effects on factional support would materialize in the second, and any gains from re-merging would only be obtainable in the third. This would complicate the analysis and notation but would not change the model's qualitative insights.

Third, in our setup, factions face no uncertainty over the consequences of a split for their electoral support, or the evolution of the electorate's ideological tastes (λ_2). We impose this assumption in order to more clearly illustrate the mechanism behind the results (excluding the possibility of mistakes), and show that dynamic incentives may generate splits in equilibrium even if factions can perfectly anticipate that this will be costly in the short run (i.e., the split is statically damaging). Importantly, however, introducing a small amount of uncertainty would not alter our qualitative conclusions. In concluding the paper, we then briefly discuss how a large amount of uncertainty may enrich our dynamics.

Finally, we assume that the effect of a split on the factions' expected valence is fully persistent, carrying through to the next period even if the factions reunite in the same party. This is to capture the intuition that the reconfiguration of electoral support that occurs due to a split may be hard to fully reverse, at least in the short run. If the split damages a faction's image and causes it to lose supporters, these supporters are hard to regain. Similarly, supporters gained will be somewhat attached to the faction after they mobilize and thus will not be completely lost even in case of a re-merger. In reality, there could be some dilution of this effect, and we could parametrize the extent to which the effect of the split "depreciates" over time, or when the party reunites. Reassuringly, our results only require a sufficient level of persistency: as long as this depreciation is low enough, our result would go through.

4. Analysis

4.1. The Voters

We begin by characterizing voters' behavior. At time t, a mass $1-\lambda_t$ of potential voters belongs to the right-wing ideological camp. A right-wing voter never supports a left-wing party, and instead chooses between voting for the right-wing party R and abstaining. The utility from voting is $v_i^R - c$, while the normalized utility from abstaining is zero. Thus, a right-wing voter votes for R if and only if $v_i^R \geq c$, and the measure of votes for R in period t is

$$N_t^R = (1-\lambda_t)\Big(1 - G_R(c)\Big). \tag{6}$$

Turning to the left-wing electorate, each voter chooses whether to support faction A, faction B, or abstain. Given our formulation, this decision rule applies both when the factions are separate parties and when they are unified.

The measure of voters supporting each faction is given by

$$N_t^A = \lambda_t \int_c^\infty g_A(x)\, G_B(x)\, dx, \qquad N_t^B = \lambda_t \int_c^\infty g_B(x)\, G_A(x)\, dx, \tag{7}$$

where g_j and G_j denote the density and CDF of the valence of faction $j \in \{A, B\}$, respectively. These expressions capture the share of voters for whom faction j provides both the highest and a positive net utility from voting. As mentioned above, recall that under a united left-wing party, each left-wing voter l votes for L if and only if $\max\{v_\ell^{L^A}, v_\ell^{L^B}\} > c$. This implies that, when the factions run together in the same party, the measure of votes for the party is given by $N_t^A + N_t^B$, as characterized above.

We can now examine how a split, which shifts factions' expected valences, affects their electoral support.

Lemma 1. *The effect of a split on the factions' electoral support need not be zero-sum. For example, if a split shifts the expected valence of both factions by the same amount δ, so that $\bar{v}^j(\Sigma = 1) = \bar{v}^j(\Sigma = 0) + \delta$ for $j \in \{A, B\}$, then both factions' support move in the same* *direction:*

$$\frac{\partial N_t^A}{\partial \delta} > 0 \quad and \quad \frac{\partial N_t^B}{\partial \delta} > 0.$$

Hence, a positive (negative) shift in expected valences increases (decreases) the support of both factions simultaneously.

Lemma 1 highlights that the impact of a split on the two factions' electoral bases is not necessarily redistributive. The clearest illustration is the case when a split uniformly improves (or worsens) both factions' appeal—for instance, by clarifying the ideological profile of the left camp or by confusing voters. In this case, each faction's support will increase (or decrease) together. This is because each faction's realized support depends on two factors: how the faction's valence compares to the cost of voting, and how the faction's valence compares to the other's. As the faction's own (expected) valence increases, the faction will be able to mobilize some of the voters that would have otherwise abstained. In addition, this increase influences the voters' decision of which faction to support. When both factions' (expected) valences increase by the same amount, the redistributed effect is a net zero, and both factions build support by mobilizing abstainers. Only when the split has asymmetric effects on expected valences will one faction gain at the expense of the other, and the overall effect may see one faction's support increase while the other decreases.

4.2. The Factions

Having characterized voter behavior, we now turn to the factions' strategic problem. We begin with a static benchmark and then examine how dynamic considerations may alter the results.

4.3. A Static Benchmark

A one-period game captures factions' immediate incentives, and provides a baseline against which to interpret the richer dynamics of the two-period setting. Recall that we denote $S_t^A(\lambda_t, \Sigma_t)$, $S_t^B(\lambda_t, \Sigma_t)$ and $S_t^R(\lambda_t, \Sigma_t)$ the period-t share of voters voting in favor of faction/party A, B and R respectively.

Proposition 1. *A split emerges if and only if*

$$f(S^A(\lambda_t, 1)) + f(S^B(\lambda_t, 1)) > f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0)). \tag{8}$$

The static benchmark therefore predicts that whether factions remain together or separate depends on the net effect of splitting on their combined support relative to the convex efficiency gains from unity. The logic is straightforward. When condition (8) does not hold, a split *destroys* surplus in period t. Either a split directly harms the camp's support, $S^A(\lambda_t, 1) + S^B(\lambda_t, 1) < S^A(\lambda_t, 0)+S^B(\lambda_t, 0)$, or the support increases but not enough to compensate for the lost efficiency premium of running together (given by the convexity of f). In this case, the factions can always find a bargaining agreement on how to divide the surplus to avoid a split. In contrast, when condition (8) holds, a split *creates* surplus. In this case, a split is inevitable: the size of the pie when the party remains together is never enough to compensate both factions for their outside option.

An implication of Proposition 1 is that a split that helps build support may or may not emerge in equilibrium of the static game depending on the broader ideological climate (the convexity of f creates an interaction effect between λ_t and the direct effect of a split on the factions' support). Shifts in the electorate can tilt the balance between efficiency gains from unity and voter gains from separation. Thus, a split that has the same positive effect on the camp's support may or may not emerge in equilibrium, depending on the ideological leaning of the electorate.

4.4. Dynamic Model

Moving to the dynamic model, we begin by characterizing the equilibrium of the second period.

Lemma 2. *Suppose there was a split in the first period. Then, factions must always re-merge in the second.*

Intuitively, once a split occurs, because the benefit of splitting does not accumulate over time, factions always re-merge in the second period to accrue the efficiency premium of staying together.

Suppose instead that factions remain together in period 1. Then, factions can initiate a split in period 2, for the exact same dynamics highlighted in Proposition 1:

Lemma 3. *Suppose there was no split in the first period. Factions split in the second period if and only if*

$$f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1)) > f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)). \tag{9}$$

The proof is identical to that of Proposition 1, highlighting that a second-period split emerges entirely for static incentives. We now move to the first-period analysis and the characterization of the equilibria of the game.

4.5. If Split is Statically Efficient

First, suppose that condition (8) holds at $t = 1$, so that a split is statically optimal in the first period. The following result establishes that, under these circumstances, a split must also arise in equilibrium of the dynamic game.

Lemma 4. *If $f(S^A(\lambda_1, 1)) + f(S^B(\lambda_1, 1)) > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$, then a split always emerges in equilibrium in the first period.*

This result follows from two observations. First, in our model, factions can freely re-merge after a split if they wish. In equilibrium, they will always choose to re-merge after splitting in period 1. Second, recall that a split is statically optimal only if it attracts voters to the ideological camp. Thus, a statically optimal split never harms the party's future performance. If a split is statically optimal in the first period, the factions can split today to reap the benefits and re-merge tomorrow to enjoy the efficiency premium of a stronger party. There is therefore no incentive to avoid or delay the split. Lemma 4 then shows that any statically optimal split will also arise in the dynamic setting, whether initiated by the proposer or reached consensually.

4.6. If Split is Statically Inefficient

We now turn to the opposite case, where a first-period split would not occur in the one-shot (static) version of the game. Formally, for the remainder of the paper we will assume that

$$f(S^A(\lambda_1, 1)) + f(S^B(\lambda_1, 1) < f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)). \tag{10}$$

Recall that this means that a split destroys surplus. This implies that there exists an allocation of the party's resources that would increase both factions' static payoff, compared to their payoff from splitting. Naturally, then, if a split emerges in equilibrium it must be due to dynamic incentives. The remainder of the analysis is dedicated to characterizing such incentives, and the conditions under which they lead to a split in the first period.

Before establishing when such damaging splits can actually emerge, we begin with a simple but important negative result: if there is no convexity in the payoff function—that is, if f is linear—then there cannot be equilibria of the dynamic game with statically inefficient splits.

Lemma 5. *Suppose f is linear. If a split is not an equilibrium of the static game at $t = 1$, then it is not an equilibrium of the dynamic game.*

The reasoning is straightforward. There are two potential reasons why a statically damaging split might emerge in equilibrium. First, a faction may attempt to increase its relative size and thereby strengthen its bargaining position in the next period, i.e., raise the probability of being recognized as the proposer. However, when f is linear, there are no rents from being recognized as the proposer. In equilibrium, the proposer must make the receiver indifferent in the second period to keep the party together. Since, under linearity, the party's resources are exactly equal to the sum of what factions could obtain independently, there is no surplus to be extracted by the proposer. Hence, no faction has an incentive to initiate a damaging split in the first period.

Second, if a split increases the camp's overall support (though not enough to be statically optimal), the factions might consider initiating it today to reap benefits tomorrow, when re-merging into a stronger party. These benefits, however, arise only when f is convex. If f is linear, there is no surplus from joining forces: in equilibrium, factions receive the same payoff in the second period whether they run together or apart. They could therefore remain merged today and split tomorrow if it proved advantageous. In other words, as above, no faction has an incentive to initiate a damaging split in the first period when f is linear.

This result suggests a simple yet neglected relationship between electoral institutions and intra-party incentives. Constitutional design scholars typically focus on the *static* incentives that institutions produce at the party level. A powerful intuition in this literature, known as Duverger's law, states that we should expect a less fragmented party system under more majoritarian electoral rules (Duverger, 1951). While this intuition is upheld in our model if factions merely consider their static payoffs, Lemma 5 highlights that dynamic considerations may generate the opposite results, whereby the disproportionality of the electoral system is precisely what opens the door to statically inefficient splits.

In this vein, our model may offer some insights as to why evidence on the empirical relevance of the Duvergerian proposition is mixed (see, e.g., Cox (1997); Lijphart (1994); Diwakar (2007); Singer (2013)). Existing scholarship explains this mixed evidence by suggesting that Duverger forces may be *dampened* when voters or parties fail to act strategically (Cox, 1997), or because of societal cleavages that interact with electoral institutions (Ordeshook and Shvetsova, 1994). In contrast, our analysis emphasizes that, even if a Duvergerian logic is statically upheld, the effect of disproportionality on the effective number of parties may go in the *opposite* direction once we consider factions' dynamic incentives.

Having established this negative result, in the remainder of the paper we will focus on the case in which $f'' > 0$, and turn to the more substantive question: under what conditions can statically inefficient splits nevertheless be sustained in equilibrium when f is convex?

When a split is statically inefficient, it is useful to distinguish two different scenarios depending on its effect on the overall size of the camp:

1. Support-reducing split: the split reduces total support for the camp

$$S^A(\lambda_t, 1) + S^B(\lambda_t, 1) < S^A(\lambda_t, 0) + S^B(\lambda_t, 0).$$

   In this case, the factions collectively lose supporters by separating.

2. Support-improving split: the split increases total support for the camp,

$$S^A(\lambda_t, 1) + S^B(\lambda_t, 1) > S^A(\lambda_t, 0) + S^B(\lambda_t, 0),$$

   but the gain is not large enough to outweigh the efficiency premium of unity *in the first period*, so that

$$f(S^A(\lambda_1, 1)) + f(S^B(\lambda_1, 1) < f((S^A(\lambda_1, 0) + S^B(\lambda_1, 0)).$$

   Here, separation expands the camp but still delivers lower payoffs in the short run.

In what follows, we also seek to determine which faction is responsible for triggering a split. Is separation always driven by one of the factions? Under what conditions might the factions instead agree to separate consensually? To formalize this, we adopt the following definition:

Definition 1. *Let $\bar{U}_t^1$ denote the lowest offer faction j is willing to accept to remain within the party at time $t = 1$. We say that a split that emerges on the equilibrium path in period $t = 1$ is*

i) Unilateral and initiated by faction j if $\bar{U}_1^{-j} < 0$,

ii) Consensual if $\bar{U}_1^j > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$ for both $j \in \{A, B\}$,

iii) Conflictual otherwise.

This categorization captures the idea that different types of splits may appear distinct to outside observers. When a split is initiated by faction j, that faction cannot plausibly shift blame: it cannot claim willingness to compromise while portraying the other side as intransigent. Here we should expect one faction to make unilateral attempts at appeasement and appeals to the other to avoid a split. By contrast, in a conflictual split, each faction can try to blame the other, as both can identify a non-zero offer they can pretend to make fully expecting the opponent to reject. Finally, in a consensual split we may observe little open conflict preceding the split and little discussion over how to avoid it, since both factions benefit from going their separate way.

Notice that, if $\bar{U}_t^j < 0$ for both factions, clearly a split cannot emerge because no faction can profit from it. Similarly, if $\bar{U}_t^{-j} < 0$ and $0 < \bar{U}_t^j < f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0))$, faction $-j$ is willing to give j the amount of resources she demands to remain within the party, so there would be no split either. Hence, for the unilateral case (i) it has to be that $\bar{U}_t^j > f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0))$ for the faction initiating the split, or else we would observe unity in equilibrium.

Case 1: Support Reducing Split. We begin with the more severe case in which the split is damaging to the camp as a whole. Abusing notation, in what follows we will use $\pi^j(\Sigma_t)$ to denote $\pi^j(\rho^j(\Sigma_t))$.

Proposition 2. *Suppose a split reduces the camp's support, $S^A(\lambda_t, 1) + S^B(\lambda_t, 1) < S^A(\lambda_t, 0) + S^B(\lambda_t, 0)$. Then, a first-period split is always unilateral. Furthermore, there exists a unique pair $\bar{\Delta} > 0$ and $\bar{\lambda} > \lambda_1$ s.t. j initiates a first-period split if and only if* *(i)* $\pi^j(1) - \pi^j(0) > \bar{\Delta}$, *and*

(ii) $\lambda_2 > \bar{\lambda}$.

Otherwise, if at least one of these conditions fail, the equilibrium features a stable merger.

The equilibrium behavior described under the conditions in Proposition 2 is the following: factions separate in the first period but re-merge in the second, consistent with Lemma 2. Thus, this result identifies an equilibrium featuring a reversal pattern: the factions break away despite the short-term inefficiency of fragmentation, only to reunite in the second period. The logic underlying this dynamic is that of a "bigger fish in a smaller pond". Even though the split depresses the camp's total support, it may affect the factions' relative size: either one faction gains at the expense of the other, or both lose supporters but at different rates. In these cases, one faction stands to gain from the split because its probability of being recognized as the proposer in the second period increases. Under the proposition's conditions, this faction initiates the split—paying a cost today and harming the party's future performance to improve its bargaining position and claim a larger share of a smaller pie.

The logic described above highlights why splits that damage support can only be unilateral. Since a split wastes resources overall, it is effectively zero-sum: one faction's dynamic gain comes at the expense of the other. If the receiver prefers to exit— even when it could be offered the whole pie — the proposer strictly prefers to remain in the party — even if this requires giving up the entire pie; conversely, if the proposer benefits from separation, the receiver strictly prefers to remain.

This unilateral logic is illustrated by the 2017 split of *Articolo 1* from Italy's Democratic Party (PD). Then-party leader Matteo Renzi openly sought to prevent the breakup: "I am appealing to the (*faction*) leaders: stop the division machine. Do not leave". He even offered the internal confrontation demanded by the minority: "I want to avoid a split: if the minority tells me, either Congress or split, I say Congress."[11] Here, "Congress" refers to convening a party congress—effectively opening the way to alter the internal balance of power and resources to appease the minority faction. Despite this, Pierluigi Bersani and his allies proceeded with the

[11] See *Sky TG24*, 17 February 2017: https://tg24.sky.it/politica/2017/02/17/pd-renzi-no-scissione-congresso-elezioni?.

split. As noted above, the split was explicitly framed as a temporary, costly expedient, with the splinters open to reunification under the right conditions. Evidence indicates that the split hurt both factions' electoral appeal: even combined, their vote shares fell short of the PD's support in prior elections. Consistent with our expectations, *Articolo 1* dissolved and rejoined the PD in 2023 from a position of greater strength; notably, the PD's current leader, Elly Schlein, is a former member of *Articolo 1*.

A similar dynamic characterized the 2008 split of the *Pro-Park Alliance* from South Korea's Grand National Party (GNP). The conflict began when ethics rule caused Park Geun-hye's loyalists to be excluded from the party's internal nomination process ahead of the legislative elections. Facing the threat of exit from pro-Park loyalists, in an attempt to prevent a rupture, the GNP revised its ethics rules to allow some pro-Park candidates to run under the party's banner.[12] Despite this concession, many of Park's supporters left the party to form a new organization named after her.[13] Park herself remained within the GNP, arguably not to damage the legitimacy of her future claims to leadership, but publicly condemning the process and boycotting campaign events for party candidates, telling her followers: "Come back after surviving."[14] Consistent with our expectations, the split led the GNP to suffer significant electoral losses.[15] The pro-Park lawmakers then used the split to strengthen their leverage: they were later readmitted under favorable terms, and the reunification paved the way for Park's rise to the presidency in 2012.

The conditions in Proposition 2 clarify when such unilateral damaging splits can arise. Condition (i) captures the effect of the split on the splinter's bargaining power—specifically, the change in the probability of being recognized as proposer in the second period, with and without a split. A split becomes possible only if this difference is sufficiently large. Substantively, this requires that the split significantly alters the relative size of the factions, producing a sharp

[12] See *Korea JoongAng Daily*, 4 February 2008: https://koreajoongangdaily.joins.com/2008/02/04/politics/Park-accepts-compromise-on-GNP-ethics-rule/2885947.html.

[13] See *Korea JoongAng Daily*, 19 March 2008: https://koreajoongangdaily.joins.com/2008/03/19/politics/Spurned-by-party-Park-loyalists-walk-out-of-GNP/2887637.html.

[14] See *The Korea Times*, 9 April 2008: https://www.koreatimes.co.kr/southkorea/20080409/pro-park-winners-seek-to-rejoin-gnp.

[15] The party experienced a major setback in the 2010 local elections, recording a severely disappointing performance. See the *BBC*, 3 June 2010: https://www.bbc.com/news/10211824

reconfiguration of support, and that the party's internal institutions are sufficiently responsive to such shifts.

Condition (ii) concerns the ideological trend, λ_2. The effect of λ_2 is not obvious: after all, the ideological trend influences the factions' payoffs whether they run together or apart. Yet, the analysis shows that for a damaging split to be sustainable, future prospects must be sufficiently favorable. If ideological trends are stable or adverse ($\lambda_2 \leq \lambda_1$), *even net of the effect of the split*, the camp's total support tomorrow will be (weakly) smaller than today. In this case, the pie that the factions bargain over shrinks over time, and they can always find an agreement on how to divide resources today that leaves both better off: compensates the would-be splinter and avoids the cost of a split. Thus, a split sustained by the "bigger fish in smaller pond" logic can never emerge when $\lambda_2 \leq \lambda_1$. Indeed, a necessary condition for such a split is that λ_2 is sufficiently large, ensuring that tomorrow's pie is valuable enough for the splinter to trade off today's static cost against the increased probability of being proposer tomorrow. This highlights a kind of "dynamic resource curse:" the expectation of abundant resources in the future can make it strategically rational to tolerate inefficiency today.

Having characterized the conditions under which damaging splits arise, we now turn to cases where splitting expands the overall support of the camp.

Case 2: Support Building Split. When a split brings in new supporters but not enough to offset the convex gains from staying together in the first period, the dynamic game allows for additional outcomes. Unlike in the damaging case, where a split can never be consensual and must always be unilateral, beneficial splits open up additional possibilities: depending on parameters, the split may be consensual, unilateral, or conflictual.

When the split brings increased support to the camp as a whole, a faction may want to initiate a split even when it does not result in an increase in its bargaining power. The dynamic gains from the split, in fact, come from the fact that an increase in the camp's support would translate into a larger efficiency premium from running together in the second period. To see this, consider the factions' tradeoff. One option is to stay together in the first period to avoid the static cost, and then either expand the camp's support tomorrow by splitting (thus giving up the efficiency premium) or remain together to exploit the efficiency (but giving up the gain from increased support). The other option is to split today, thus incurring the static cost, and then re-merge tomorrow thus enjoining *both* the increased support *and* the efficiency premium. If the dynamic gain from the second option is sufficiently large, a split will emerge in equilibrium, and may even be initiated by the faction that stands to lose in terms of relative bargaining power. Of course, there may be even stronger incentives from the faction that stands to gain an increase in bargaining power, but the above logic explains why, in contrast with Proposition 2, a split here may be consensual.

Formally, the analysis must consider the two possible equilibrium outcomes in the second period. First, suppose that $f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1)) < f((S^A(\lambda_2, 0) + S^B(\lambda_2, 0))$. That is, despite the fact that a split increases the camp's support, it is not statically optimal in the first *nor* the second period.

Proposition 3. *Suppose a split is beneficial to the camp as whole, but the effect is not too large:* $S^A(\lambda_t, 0) + S^B(\lambda_t, 0) < S^A(\lambda_t, 1) + S^B(\lambda_t, 1)$ *and* $f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1)) < f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0))$. *There exists a unique* $\hat{\lambda}$ *such that a first-period split emerges in equilibrium if and only if* $\lambda_2 > \hat{\lambda}$. *The split can be consensual, unilateral or conflictual. Otherwise, if* $\lambda_2 < \hat{\lambda}$*, then the equilibrium features a stable merger.*

Proposition 3 shows that when the efficiency premium still dominates in both periods, the factions prefer unity unless the ideological trend is sufficiently favorable. If λ_2 is sufficiently large, then the prospect of a large pie tomorrow sustains a temporary split today, which may be consensual or initiated by either side depending on relative bargaining power. By contrast, when λ_2 is low, ideology-driven gains to the camp cannot justify the short-run losses from separation, and the equilibrium outcome is a stable merger.

Finally, suppose that splitting generates large enough gains to outweigh the efficiency premium in the second period: i.e., $f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1)) > f((S^A(\lambda_2, 0) + S^B(\lambda_2, 0))$. In this case, the dynamic and static logics converge: separation becomes unavoidable. The ideological trend however determines whether the split emerges immediately, or is deferred until the second period.

Proposition 4. *Suppose a split has a large positive effect on the camp as a whole:* $f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) < f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1))$. *Then, there exist unique* $\tilde{\lambda}^p < \lambda_1$ *and* $\tilde{\lambda} > \tilde{\lambda}^p$ *s.t.*

- *When $\lambda_2 < \tilde{\lambda}^p$, in equilibrium the party remains merged in the first period and splits in the second,*

- *When $\lambda_2 \in (\tilde{\lambda}^p, \tilde{\lambda})$, a conflictual split emerges in the first period, and*

- *When $\lambda_2 > \tilde{\lambda}$, a consensual split emerges in the first period.*

When the gains from splitting are large, the outcome depends on how favorable the ideological trend is. For low λ_2, the party delays fragmentation, remaining united in the first period before splitting in the second. For intermediate λ_2, a split occurs immediately, driven by the proposer who finds it too costly to retain the receiver. And when λ_2 is very high, unity cannot be sustained, and the factions separate consensually from the start. The logic underlying a consensual split is that of a "smaller fish in a bigger pond": one of the factions is willing to pay a cost today *and* see its future bargaining position within the party worsen, in order to allow the camp to consolidate support. The anticipated future re-merger provides the appropriate incentives, with this faction gaining from obtaining a smaller share of a bigger pie.

An example of a support-building, consensual split sustained by the "bigger pond" logic is the creation of *Thjodvaki* from Iceland's People's Party in 1994. The new party, led by Johanna Sigurdardottir, explicitly sought to unify the Icelandic center-left. The disagreement with the parent party was limited: the two groups continued to cooperate closely, even running joint lists in several local coalitions. This coordination reveals that the separation was not driven by conflict but by a shared strategic goal of expanding the social-democratic camp's overall appeal. The party's official history later described *Thjodvaki* as "founded in 1994 with the stated goal of unifying Icelandic social democratic parties."[16] The split thus represented a consensual attempt to enlarge the ideological camp rather than a struggle for internal power. The two organizations eventually re-merged into a larger center-left formation, the Social Democratic Alliance.

A similar dynamic appears to be unfolding in Greece with the recent exit of former Prime Minister Alexis Tsipras from *Syriza*. Upon announcing his departure, Tsipras told his colleagues:

[16] See the historical summary of the Social Democratic Alliance: https://xs.is/sogulegt-agrip-flokksins. The same cooperative intent was echoed in *Thjodvakabladid* (March 20, 1996), which argued that "It requires a powerful social-democratic movement; social-democratic organizations should unite:" https://timarit.is/page/3646355.

"We will not be rivals. And perhaps soon we will travel together again to more beautiful seas."[17] This rhetoric of future cooperation suggests a form of collusive separation, aimed at strengthening the broader left rather than producing an enduring rupture. Consistent with this interpretation, Syriza's current leader, Socrates Famellos, reacted respectfully, stressing that although they hold "different perspectives on how to get rid of the government of Kyriakos Mitsotakis and his centre-right New Democracy party, we will not be opponents."[18] As with the Icelandic case, the move appears consensual and guided by a "bigger pond" logic: political observers in Athens expect Tsipras to pursue a more centrist, progressive orientation, distancing himself from Syriza's hard-left profile, a move that would allow both parties to more effectively mobilize different segments of the electorate and thus increase the camp's total base.[19]

These two examples stand in stark contrast with the cases presented in the previous section — Italy's *Articolo 1* and Korea's Pro-Park split — which were marked by open conflict, explicit attempts at intra-party appeasement to avoid division, and a bargaining logic driven by internal power dynamics rather than the broader camp's electoral success.

Taken together, Propositions 2–4 fully characterize the equilibrium dynamics of the model. Damaging splits can arise only under strict conditions and are always unilateral. Beneficial but insufficient splits may occur if the future ideological trend is favorable enough, with the initiating side depending on bargaining considerations. When the gains from separation are large, splits are unavoidable, with timing and initiator determined by the ideological environment.

5. Extensions and Robustness

Before moving to analyzing the model's comparative statics, we discuss the robustness of our results to relaxing some of our model's assumptions.

In our model, we focus on a world that is essentially frictionless: factions are always free to re-merge in the same party if they wish to do so, there are no restrictions to the internal bargaining

[17]See *Euronews*, 7 October 2025: https://www.euronews.com/2025/10/07/former-greek-pm-tsipras-quits-parliament-amid-new-party-speculation.

[18]See *Balkan Insight*, 8 October 2025: https://balkaninsight.com/2025/10/08/greeces-leftist-ex-pm-alexis-tsipras-leaps-into-political-unknown/.

[19]See *Euractiv*, 8 October 2025: https://www.euractiv.com/news/former-greek-pm-tsipras-resigns-from-parliament-fuels-new-party-speculations/.

process, and no uncertainty over the consequences of a split or the ideological trends in the electorate. Our results then demonstrate that, even absent such frictions, statically inefficient splits may occur. Here we discuss how introducing these frictions affects the sustainability of our results— some weaken the conditions for dynamic splits, while others reinforce them.

Credible commitment to re-merge. First, one may imagine that the opportunity for splintering factions to re-merge and form a viable united party may require the appropriate political conditions to materialize. For example, the factions may lack the material resources needed for a merger, or unanticipated political scandals may make the merger non-viable. In our model, we can capture these frictions by assuming that, in period $t = 2$, the game can either be in state $\omega = m$, or in state $\omega = \emptyset$, where $\omega = m$ denotes the state where the merger is possible, i.e., factions are free to merge if they both agree. Let p be the probability that $\omega = m$, then we have

Proposition 5. *There exists a $\bar{p} > 0$ such that a statically inefficient split never emerges in equilibrium when $p < \bar{p}$.*

This results is intuitive, but highlights an important property of the dynamic incentives we uncover. In the baseline model, a statically inefficient split emerges in equilibrium when the factions anticipate sufficient gains, to be realized tomorrow by re-merging in a united party. In that model, a commitment to re-merge is always credible because of the efficiency gains from unity (i.e., the convexity of f). Proposition 5 then highlights that, in a world with frictions that may render the anticipation of a re-merger less credible, a statically inefficient split only emerges if these frictions are sufficiently small.

Ego rents and the indivisibility of the party's spoils. Second, our baseline model assumes no restrictions on how factions divide the spoils: the proposer can, in principle, offer the entire pie to the other faction. This ignores the possibility that some spoils are indivisible and directly attached to the leading faction (the proposer). For instance, beyond the division of resources, leadership roles carry ego rents that the proposer may be unable to cede, especially if the party constitution regulates leadership selection.

In our model, we can capture this observation by assuming that there is an upper bound $\bar{x} < 1$ to the share of resources that the proposer can offer to the veto player in each period. Mechanically, this can generate inefficient splits in equilibrium even in the static setting, because the factions may be willing but unable to implement a Pareto-efficient allocation. When we consider the factions' dynamic incentives, this indivisibility increases the value of being recognized as the proposer in the second period, and thus increases the benefit (cost) of a split for the faction whose relative size would increase (decrease). Intuitively, then, the effect of this indivisibility mirrors the comparative statics on the party's internal institution we describe in the next section. First, suppose the split damages the camp's overall support, and is thus sustained by a "bigger fish in a smaller pond" logic. As we established above, this split is always unilateral, and emerges in the baseline model despite the 'losing' faction's willingness to offer the entire pie to the splinter. Then, the presence of the upper bound $\bar{x}$ can only make a split easier to sustain, by increasing the gains for the faction that can capitalize on the split to consolidate its bargaining position. In contrast, the effect may go in the opposite direction when the split is statically inefficient but helps build support for the camp. In this case, the split can become harder to sustain, as the indivisibility increases the 'losing' faction's willingness to concede in the first period to avoid the dynamic cost.

Uncertainty. Finally, to avoid the possibility of inefficient splits being the result of a strategic mistake, in the baseline model we assume that the factions can perfectly anticipate the consequences of a split (i.e., they know δ_A and δ_B), as well as the ideological trends in the electorate (i.e., λ_1 and λ_2). In our model, payoffs are continuous in these parameters, therefore it is intuitive that uncertainty would not change the qualitative results, as long as it is not too large. Indeed, uncertainty may even generate inefficient splits where none would emerge in the baseline model, for example if the expected value of λ_2 is below the relevant cutoffs but the variance is large enough that the chances of its realization being above the cutoff is significant.

More interesting, however, is to consider how the players' strategic incentives may change if uncertainty is coupled with heterogeneous priors, especially on the effects of a split. Suppose for example that a faction is convinced that its outside option, i.e., the expected success from running alone, is better than the opponent recognizes. Here, a faction may initiate a split to demonstrate its strength and improve its future bargaining position as a potential veto player. Such a split would be temporary and may even be statically inefficient when the expected increase in the faction's support does not offset the efficiency premium from running together. Analogous to the "bigger fish in a smaller pond" logic, this outcome arises under a sufficiently large λ_2 and a sufficiently large mismatch in the factions' beliefs about the net effect of a split. This dynamic echoes results in the conflict literature, where inefficient war can emerge from private information about the parties' strength (Fearon, 1995*b*).

5.1. Comparative Statics: Intra-Party Institutions

We now leverage our theoretical results to study how institutional features regulating competition within parties influence party stability and fragmentation. A prominent argument in the literature is that increasing intra-party power sharing should strengthen party unity. For instance, introducing primaries may build consensus among party members and legitimize the chosen candidate in the eyes of those not selected. We show, however, that this intuition only holds under certain conditions, depending on how a split affects the total support of the camp.

How do we capture the internal organization of a party? In our model, the factions' relative support ρ^j affects their recognition probability π^j—and therefore their power within the party. In what follows we will impose the following functional form for the recognition probability:

$$\pi^j(\rho^j) = \frac{1}{2} + \phi\rho^j(\Sigma), \tag{11}$$

where the parameter ϕ is appropriately bounded to ensure the probability of recognition is between zero and one. In a fully egalitarian institution, $\pi^j = 1/2$ regardless of factions' relative strength. By contrast, in a less egalitarian party, the elasticity of π^j with respect to ρ^j is high: as factions' relative support changes, so does their bargaining power. Thus, we adopt the following definition:

Definition 2. *Let $\eta = \frac{\partial \pi^j}{\partial \rho^j}$. For any two $\eta' > \eta'' > 0$, we say that the party organization is more egalitarian under η' than under η''.*

Clearly, given our functional form, the parameter ϕ captures the elasticity of recognition probability with respect to factional support. Lower values of elasticity (higher values of ϕ) correspond to a less egalitarian party organization. The next result shows how this parameter affects party stability.

We first consider the case of splits that reduce the camp's total support.

Proposition 6. *Suppose that the split reduces the camp's total support:* $S^A(\lambda_t, 1) + S^B(\lambda_t, 1) < S^A(\lambda_t, 0) + S^B(\lambda_t, 0)$. *Then, making the party organization more egalitarian weakly decreases the likelihood of a split (in the sense of set inclusion).*

Recall that, from Proposition 2, when a split decreases the camp's support it emerges unilaterally in equilibrium if and only if there exist a player j for which the difference $\pi^j(1) - \pi^j(0)$ is positive and sufficiently large. Thus, fixing the effect of a split on the factions' relative support, making the party more egalitarian (thus reducing the elasticity of π^j) can only make this condition harder to sustain. Intuitively, the incentive to split comes from a faction's desire to strengthen its relative standing within the party. As $\pi^j(1) - \pi^j(0)$ increases, these incentives become stronger, making damaging splits more sustainable and party unity harder to preserve.

Proposition 6, however, does not imply that egalitarian institutions always promote party stability. Whether power sharing prevents fragmentation crucially depends on the effect of the split on total support. Accordingly, the next result shows that egalitarian institutions can be harmful to party stability when splits are beneficial (yet statically inefficient in both periods).

Proposition 7. *Suppose the split is beneficial to the camp as whole:* $S^A(\lambda_t, 0) + S^B(\lambda_t, 0) < S^A(\lambda_t, 1) + S^B(\lambda_t, 1)$. *Then, there exist parameter values under which making the part organization more egalitarian increases the likelihood of a split (in the sense of set inclusion).*

The intuition is straightforward. When the split helps build support for the ideological camp, a faction may be willing to give up *both* first-period payoff *and* future bargaining power in order to strengthen the camp. However, the cost is higher when the party is less egalitarian, as in this case bargaining power is more valuable. Thus, a less egalitarian structure makes the faction less willing to 'take one for the team', and the incentives to support a split weaken.

Overall, these results help explain why institutional reforms such as primaries can sometimes stabilize parties by reducing damaging splits, yet in other cases exacerbate fragmentation when splits expand the camp. In this sense, they align with the heterogeneity observed in different empirical settings. For instance, scholars specializing in Southern U.S. politics have argued that Democrats endorsed primaries to maintain their one-party dominance by averting factional defections (Key, 1949). Conversely, other researchers have posited that introducing primaries might actually exacerbate internal conflicts (Burden, 2004).

6. Conclusion

Most party systems frequently witness significant political changes, with splits and mergers of political parties taking center stage. This has led to a growing interest among scholars and political observers in understanding the complex dynamics of party politics and factionalism. This paper develops a theory to explain why factions belonging to the same party might choose to split, and when instead we should expect party unity.

Our analysis shows that the evolution of political parties cannot be understood through static logic alone. In a one-shot world, factions split only when separation increases their total support enough to offset the efficiency premium of unity. Yet once we allow factions to look ahead, the calculus changes: temporary fragmentation may arise even when it destroys value in the short run.

Two distinct dynamics illustrate this logic. According to the *bigger fish in a smaller pond* logic, a faction splits precisely because doing so weakens its rival even more, thereby improving its future bargaining position once the party reunites. In contrast, the *smaller fish in a bigger pond* equilibrium captures cooperative cycles in which factions willingly incur present costs to expand their shared ideological camp, anticipating a re-merger into a stronger, more competitive party. Both dynamics highlight that parties can fragment and recombine not out of error or conflict, but as part of a calculated strategy of political cultivation.

These dynamic forces also lead us to reconsider familiar comparative statics. Institutions that increase the efficiency premium of unity such as disproprortional electoral systems — which in static settings should deter fragmentation — can, in the dynamic game, make splitting more attractive by magnifying future rewards. Analogously, more egalitarian internal rules can either stabilize or destabilize parties depending on which logic guides the cycle.

Our model predominantly concentrates on factions within parties, but its insights can be extrapolated to pre-electoral coalitions. Analogous to our model, the decision to exit a coalition can either fortify or diminish a party's electoral support. Our model's focus on dynamic incentives faced by coalition partners then offers valuable insights for understanding the frequent formation and dissolution of alliances in multi-party systems.

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Appendix

Proof of Lemma 1. Starting from

$$N_t^A = \lambda_t \int_c^\infty g_A(x) G_B(x)\, dx, \qquad N_t^B = \lambda_t \int_c^\infty g_B(x) G_A(x)\, dx,$$

suppose a split shifts both factions' mean valences by δ. Using the change of variable $\tau = x - \delta$, we obtain

$$N_t^A(\delta) = \lambda_t \int_{c-\delta}^\infty g_A(\tau) G_B(\tau)\, d\tau, \qquad N_t^B(\delta) = \lambda_t \int_{c-\delta}^\infty g_B(\tau) G_A(\tau)\, d\tau.$$

Let $s_A(\tau) = g_A(\tau)G_B(\tau)$ and $s_B(\tau) = g_B(\tau)G_A(\tau)$. Applying Leibniz's rule,

$$\frac{\partial N_t^A}{\partial \delta} = \lambda_t \int_{c-\delta}^\infty \frac{\partial s_A(\tau)}{\partial \delta}\, d\tau = \lambda_t s_A(c-\delta) > 0,$$

and analogously $\frac{\partial N_t^B}{\partial \delta} = \lambda_t s_B(c-\delta) > 0$, since $\partial s_A/\partial\delta = \partial s_B/\partial\delta = 0$ and $s_A, s_B > 0$. $\square$

Proof of Proposition 1. Suppose the parties are together at the beginning of period t. If factions did not split in the previous period, a split is inevitable when the receiver is willing to split even if they are offered the whole pie:

$$f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0)) - f(S^A(\lambda_t, 1)) < 0 \tag{12}$$

when A is the receiver, and

$$f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0)) - f(S^B(\lambda_t, 1)) < 0 \tag{13}$$

when B is the receiver.

Next, suppose instead that the above conditions fail. This implies that, whoever is selected as the proposer can find an allocation that makes the receiver indifferent and keeps the party together. Thus, if there is a split it must be initiated by the proposer. In this case, a split occurs whenever

$$f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0)) - f(S^A(\lambda_t, 1)) < f(S^B(\lambda_t, 1)) \tag{14}$$

if B is the proposer. If A is the proposer, a split instead occurs if

$$f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0)) - f(S^B(\lambda_t, 1)) < f(S^A(\lambda_t, 1)) \tag{15}$$

Of course, both conditions reduce to

$$f(S^A(\lambda_t, 1)) + f(S^B(\lambda_t, 1)) > f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0)) \tag{16}$$

Putting all the above together, we have that a split occurs if

1. $f(S^A(\lambda_t, 1)) + f(S^B(\lambda_t, 1)) > f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0))$, or $f(S^B(\lambda_t, 1)) > f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0))$, when A is selected as the proposer, or

2. $f(S^A(\lambda_t, 1)) + f(S^B(\lambda_t, 1)) > f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0))$, or $f(S^A(\lambda_t, 1)) > f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0))$ when B selected as the proposer.

Otherwise, if both relevant conditions fail, factions remain merged.

Notice that the condition for the receiver to want a split always fails if the condition for the proposer to want a split fails. Thus, a split in period 2 occurs if and only if

$$f(S^A(\lambda_t, 1)) + f(S^B(\lambda_t, 1)) - f(S^A(\lambda_t, 0) + S^B(\lambda_t, 0)) > 0. \tag{17}$$

$\square$

Dynamic Model — Preliminaries.

Remark 1. *The effect of a split on each faction's number of votes is time-independent:*

$$\frac{N^j(\lambda_1, 1)}{N^j(\lambda_1, 0)} = \frac{N^j(\lambda_2, 1)}{N^j(\lambda_2, 0)}, \tag{18}$$

for $j \in \{A, B\}$.

Proof. First, it will be useful to introduce some notation. Notice that, for any pair $(N^j(\lambda_t, 0), N^j(\lambda_t, 1))$, we can find a $\sigma^j(\lambda_t) > 0$ s.t. we can write

$$N^j(\lambda_t, 1) = \sigma^j(\lambda_t) N^j(\lambda_t, 0). \tag{19}$$

For any $N^j(\lambda_t, 0)$ and $N^j(\lambda_t, 1)$, (19) then defines the effect of splitting. Further, notice that we can write $N^j(\lambda_2, 1) = k(\lambda_2) N^j(\lambda_1, 1)$, where $k(\lambda_2) = \frac{\lambda_2}{\lambda_1}$. Thus, it must be the case that

$$\sigma^j(\lambda_2) N^j(\lambda_2, 0) = k(\lambda_2) N^j(\lambda_1, 1) = k(\lambda_2)\sigma^j(\lambda_1) N^j(\lambda_1, 0) = \sigma^j(\lambda_1) N^j(\lambda_2, 0),$$

which obviously implies $\sigma^j(\lambda_2) = \sigma^j(\lambda_1) = \sigma^j$. □

Remark 2. *If total left-wing votes increase after a split, so that $N^A(\lambda_t, 1) + N^B(\lambda_t, 1) > N^A(\lambda_t, 0) + N^B(\lambda_t, 0)$ for $t \in \{1, 2\}$, then the combined vote shares of the two factions in both periods also increase:*

$$S^A(\lambda_t, 1) + S^B(\lambda_t, 1) > S^A(\lambda_t, 0) + S^B(\lambda_t, 0) \quad \textit{for } t \in \{1, 2\}.$$

Conversely, if total votes decrease after the split, the combined shares decrease as well.

Proof. Recall that $S^j_t = \frac{N^j_t}{N^A_t + N^B_t + N^R_t}$. Write $N^A_1(\lambda_1, \Sigma) + N^B_1(\lambda_1, \Sigma) + N^R_1(\lambda_1, \Sigma)$ as $TOT(\lambda_1, \Sigma)$.

Then

$$\frac{N^A_1(\lambda_1, 1) + N^B_1(\lambda_1, 1)}{N^A_1(\lambda_1, 1) + N^B_1(\lambda_1, 1) + N^R_1(\lambda_1, 1)} > \frac{N^A_1(\lambda_1, 0) + N^B_1(\lambda_1, 0)}{N^A_1(\lambda_1, 0) + N^B_1(\lambda_1, 0) + N^R_1(\lambda_1, 0)} \tag{20}$$

Can be rearranged as

$$\Big(N^A_1(\lambda_1, 1) + N^B_1(\lambda_1, 1)\Big) \times TOT(\lambda_1, 0) > \Big(N^A_1(\lambda_1, 0) + N^B_1(\lambda_1, 0)\Big) \times TOT(\lambda_1, 1) \tag{21}$$

and

$$\Big(TOT(\lambda_1, 1) - N^R(\lambda_1, 1)\Big) \times TOT(\lambda_1, 0) > \Big(TOT(\lambda_1, 0) - N^R(\lambda_1, 0)\Big) \times TOT(\lambda_1, 1) \tag{22}$$

Recall that a split does not influence the number of votes for the right-wing party, thus $N^R(\lambda_1, 1) = N^R(\lambda_1, 0)$. Therefore, the above simplifies to

$$TOT(\lambda_1, 0) < TOT(\lambda_1, 1), \tag{23}$$

Which is true iff $N^A(\lambda_1, 1) + N^B(\lambda_1, 1) > N^A(\lambda_1, 0) + N^B(\lambda_1, 0)$.

Given Remark 1, $N^A(\lambda_1, 1)+N^B(\lambda_1, 1) > N^A(\lambda_1, 0)+N^B(\lambda_1, 0) \iff N^A(\lambda_2, 1)+N^B(\lambda_2, 1) > N^A(\lambda_2, 0) + N^B(\lambda_2, 0)$. This concludes the proof. □

Suppose A is recognized as the proposer in the first period, and denote $\bar{U}^B$ the minimum offer B is willing to accept to stay in the party in period 1. We must consider two cases: 1) a merger in the first period remains stable in the second ($f(S^A(\lambda_2, 0)+S^B(\lambda_2, 0)) > f(S^A(\lambda_2, 1))+f(S^B(\lambda_2, 1))$), or 2) a merger in the first period results in a split in the second ($f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) < f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1))$).

We now proceed with the analysis. Given the equilibrium of the second period, the on-path factional behavior may take one of three forms: Stable Merger Equilibrium, Split-Merger Equilibrium or Merger-Split Equilibrium. Thus, in the proofs below we will always consider two cases: condition (9) fails, or condition (9) is satisfied.

Case 1: $f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) > f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1))$

In this case, if there is a merger in the first period, it is always stable in the second. Furthermore, we know that if there is a split in the first period it is always followed by a merger in the second.

Let $\Gamma_d(\Sigma, \Sigma') = f(S^A(\lambda, \Sigma) + S^B(\lambda, \Sigma)) - f(S^A(\lambda, \Sigma')) - f(S^B(\lambda, \Sigma'))$. Given the equilibrium of the second period, $\bar{U}_{stable}$ satisfies

$$\bar{U}^B_{stable} + \pi^B(0)\Gamma_{\lambda_2}(0, 1) = f(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1, 1) \tag{24}$$

which yields

$$\bar{U}^B_{stable} = f(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1,1) - \pi^B(0)\Gamma_{\lambda_2}(0,1) \tag{25}$$

Here, a first-period split emerges if either one of these sets of conditions is satisfied

1. $\bar{U}^B_{stable} < 0$, but $f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) + \pi^A(0)\Gamma_{\lambda_2}(0,1)) < f(S^A(\lambda_1, 1)) + \pi^A(1)\Gamma_{\lambda_2}(1,1)$, OR

2. $\bar{U}^B_{stable} \in (0, f(S^A(\lambda_1, 0)+S^B(\lambda_1, 0)))$, but $f(S^A(\lambda_1, 0)+S^B(\lambda_1, 0))-\bar{U}^B_{stable}+\pi^A(0)\Gamma_{\lambda_2}(0,1)) < f(S^A(\lambda_1, 1)) + \pi^A(1)\Gamma_{\lambda_2}(1,1)$, OR

3. $\bar{U}^B_{stable} > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$.

Case 2: $f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) < f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1))$

Given the equilibrium of the second period, $\bar{U}_{split}$ satisfies

$$\begin{aligned} &\bar{U}_{split} + f(S^B(\lambda_2, 1)) = \\ f(S^B(\lambda_1, 1)) + \pi^B(1)(f(S^A(\lambda_2, 1) + S^B(\lambda_2, 1) - f(S^A(\lambda_2, 1))) &+ (1-\pi^B(0))f(S^B(\lambda_2, 1)) \end{aligned} \tag{26}$$

This rearranges to

$$\bar{U}_{split} = \tag{27}$$

$$f(S^B(\lambda_1, 1)) + \pi^B(1)(f(S^A(\lambda_2, 1) + S^B(\lambda_2, 1) - f(S^A(\lambda_2, 1))) - \pi^B(0)f(S^B(\lambda_2, 1)) \tag{28}$$

which can be rewritten as

$$\bar{U}_{split} = f(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1,1). \tag{29}$$

Notice that, given the convexity of f, this is always a positive quantity.

A first-period split emerges if either one of these sets of conditions is satisfied:

1. $\bar{U}^B_{split} < f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$, but $f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) - \bar{U}^B_{split} < f(S^A(\lambda_1, 1)) + \pi^A(1)\Gamma_{\lambda_2}(1, 1)$, OR

2. $\bar{U}^B_{split} > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$.

Dynamic Model — Main Results.

Proof of Lemma 4. We want to show that a split in the first period (followed by a merger in the second) is an equilibrium of the dynamic game.

CASE 1: Let $f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) < f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1))$, i.e., $\Gamma_{\lambda_2}(0, 1) < 0$, therefore a merger in the first period would always result in a split in the second.

Suppose that A is recognized as a proposer in the first period. Faction B accepts any offer x^B_1 such that

$$U^B_1(x^B_1) + f(S^B(\lambda_2, 1)) >$$
$$f(S^B(\lambda_1, 1)) + \pi^B(1)\Big[f(S^A(\lambda_2, 1) + S^B(\lambda_2, 1)) - f(S^A(\lambda_2, 1))\Big] + (1 - \pi^B(1))f(S^B(\lambda_2, 1)).$$

Denote $\bar{U}^B$ the value of U^B that solves the above with equality, that is:

$$\begin{aligned}\bar{U}^B :=& f(S^B(\lambda_1, 1)) + \pi^B(1)\Big[f(S^A(\lambda_2, 1) + S^B(\lambda_2, 1)) - f(S^A(\lambda_2, 1)) - f(S^B(\lambda_2, 1))\Big]\\ =& f(S^B(\lambda_1, 1)) + (1 - \pi^A(1))\Gamma_{\lambda_2}(1, 1),\end{aligned}$$

where notice that $\bar{U}^B > 0$ by convexity of $f(\cdot)$. Whenever $\bar{U}^B > f(S^A(\lambda_1, 0)+S^B(\lambda_1, 0))$, a split is inevitable as the receiver rejects any offer. Suppose instead that $\bar{U}^B \in \big(0, f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))\big)$, so that a split must be initiated by the proposer if it occurs. Substituting the value of $\bar{U}^B$ into A's problem, the proposer A wants to trigger a split in the first period if and only if

$$f(S^A(\lambda_1, 1)) + \pi^A(1)\Big[f(S^A(\lambda_2, 1) + S^B(\lambda_2, 1)) - f(S^B(\lambda_2, 1))\Big] + (1 - \pi^A(1))f(S^A(\lambda_2, 1)) >$$
$$f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) - f(S^B(\lambda_1, 1)) - (1 - \pi^A(1))\Gamma_{\lambda_2}(1, 1) + f(S^A(\lambda_2, 1)$$

which simplifies to:

$$\Gamma_{\lambda_2}(1,1) > \Gamma_{\lambda_1}(0,1). \tag{30}$$

Notice that the LHS of (30) is positive because of convexity and under our assumption that the split is an equilibrium of the static game at $t=1$ the RHS is negative. Hence, condition (30) is always satisfied.

CASE 2: Let $f(S^A(\lambda_2,0)+S^B(\lambda_2,0)) > f(S^A(\lambda_2,1))+f(S^B(\lambda_2,1))$, i.e., $\Gamma_{\lambda_2}(0,1)>0$, so that a merger in the first period remains stable in the second.

We then compare a split-merger equilibrium to a stable merger equilibrium.

Suppose that A is recognized as a proposer in the first period. Faction B accepts any offer x_1^B such that

$$U_1^B(x_1^B) + \pi^B(0)\Gamma_{\lambda_2}(0,1) > f(S^B(\lambda_1,1)) + \pi^B(1)\Gamma_{\lambda_2}(1,1).$$

Denote $\bar{U}^B$ the value of U^B that solves the above with equality, that is:

$$\bar{U}^B := f(S^B(\lambda_1,1)) + \pi^B(1)\Gamma_{\lambda_2}(1,1) - \pi^B(0)\Gamma_{\lambda_2}(0,1).$$

A split occurs whenever $\bar{U}^B > f(S^A(\lambda_1,0)+S^B(\lambda_1,0))$. Suppose instead that $\bar{U}^B < f(S^A(\lambda_1,0)+S^B(\lambda_1,0))$, so that a split must be initiated by the proposer if it occurs.

First, consider the case $\bar{U}^B > 0$. A initiates a split iff

$$f(S^A(\lambda_1,1)+\pi^A(1)\Gamma_{\lambda_2}(1,1) > f(S^A(\lambda_1,0)+S^B(\lambda_1,0)) - \bar{U}^B + \pi^A(0)\Gamma_{\lambda_2}(0,1) \tag{31}$$

which simplifies to

$$\Gamma_{\lambda_2}(1,1) - \Gamma_{\lambda_2}(0,1) > \Gamma_{\lambda_1}(0,1).$$

By assumption, the RHS is negative. Thus, sufficient condition for the inequality to hold is that

$$f(S^A(\lambda_2,1)+S^B(\lambda_2,1)) > f(S^A(\lambda_2,0)+S^B(\lambda_2,0)). \tag{32}$$

which, given Remark 2, is always satisfied when the split is statically optimal at $t = 1$

Second, consider the case $\bar{U}^B < 0$. A initiates a split iff

$$f(S^A(\lambda_1, 1)) + \pi^A(1)\Gamma_{\lambda_2}(1, 1) > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) + \pi^A(0)\Gamma_{\lambda_2}(0, 1)$$

$$\pi^A(1)\Gamma_{\lambda_2}(1, 1) - \pi^A(0)\Gamma_{\lambda_2}(0, 1) > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) - f(S^A(\lambda_1, 1))$$

which can be written as

$$\pi^A(1)\Gamma_{\lambda_2}(1, 1) - \pi^A(0)\Gamma_{\lambda_2}(0, 1) > \Gamma_{\lambda_1}(0, 1) + f(S^B(\lambda_1, 1)).$$

Recall that in this case we have $\bar{U}^B < 0$, i.e.,

$$f(S^B(\lambda_1, 1)) + (1 - \pi^A(1))\Gamma_{\lambda_2}(1, 1) - (1 - \pi^A(0))\Gamma_{\lambda_2}(0, 1) < 0 \tag{33}$$

which also requires $\pi^A(1) > \pi^A(0)$. Substituting the upper bound on $f(S^B(\lambda_1, 1))$ into the proposer's condition, a split always emerges if

$$\Gamma_{\lambda_2}(1, 1) - \Gamma_{\lambda_2}(0, 1) - \Gamma_{\lambda_1}(0, 1) > 0,$$

which is always satisfied since$\Gamma_{\lambda_2}(1, 1) - \Gamma_{\lambda_2}(0, 1) > 0$ and $\Gamma_{\lambda_1}(0, 1) < 0$. □

Proof or Lemma 5. First, consider case 1: $f(S^A(\lambda_2, 0)+S^B(\lambda_2, 0)) > f(S^A(\lambda_2, 1))+f(S^B(\lambda_2, 1))$.

Here, a first-period split emerges if either one of these sets of conditions is satisfied

1. $\bar{U}^B_{stable} < 0$, but $f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) + \pi^A(0)\Gamma_{\lambda_2}(0, 1)) < f(S^A(\lambda_1, 1)) + \pi^A(1)\Gamma_{\lambda_2}(1, 1)$, OR
2. $\bar{U}^B_{stable} \in (0, f(S^A(\lambda_1, 0)+S^B(\lambda_1, 0)))$, but $f(S^A(\lambda_1, 0)+S^B(\lambda_1, 0))-\bar{U}^B_{stable}+\pi^A(0)\Gamma_{\lambda_2}(0, 1)) < f(S^A(\lambda_1, 1)) + \pi^A(1)\Gamma_{\lambda_2}(1, 1)$, OR
3. $\bar{U}^B_{stable} > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$.

Suppose $f'' = 0$, i.e., f is linear. Notice that this implies $\Gamma_d(1,1) = 0$. Thus, the conditions for a split become

1. $f(S^B(\lambda_1, 1)) - \pi^B(0)\Gamma_{\lambda_2}(0, 1) < 0$, but $\Gamma_{\lambda_1}(0, 1) + f(S^B(0, 1)) + \pi^A(0)\Gamma_{\lambda_2}(0, 1)) < 0$, OR

2. $f(S^B(\lambda_1, 1)) - \pi^B(0)\Gamma_{\lambda_2}(0, 1) \in (0, f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)))$, but $f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) - \bar{U}^B_{stable} + \pi^A(0)\Gamma_{\lambda_2}(0, 1)) < f(S^A(\lambda_1, 1))$, OR

3. $\bar{f}(S^B(\lambda_1, 1)) - \pi^B(0)\Gamma_{\lambda_2}(0, 1) > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$.

$\Gamma_\lambda(0, 1) > 0$ by assumption. Thus, the first set of conditions cannot be satisfied. Plugging in the value of $\bar{U}$, the second set of conditions requires $\Gamma_{\lambda_1}(0, 1) + \Gamma_{\lambda_2}(0, 1) < 0$, which cannot be satisfied. The third condition can be rewritten as $-\pi^B(0)\Gamma_{\lambda_2}(0, 1) > \Gamma_{\lambda_1}(0, 1) + f(S^B(\lambda_1, 1))$, and thus cannot be satisfied.

Thus, in this case a split cannot emerge in the first period when f is linear.

Next, consider the second case: $f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) < f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1))$. Supposing f is linear, the conditions for a split become

1. $f(S^B(\lambda_1, 1)) < f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$, but $f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) - f(S^B(\lambda_1, 1)) < f(S^A(\lambda_1, 1))$, OR

2. $f(S^B(\lambda_1, 1)) > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$.

Neither of these conditions can be satisfied when a split is not an equilibrium of the static game, as this requires $f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) > f(S^B(\lambda_1, 1)) + f(S^A(\lambda_1, 1))$. □

Proof of Proposition 2. Here, we suppose that the split damages the support of the camp as a whole. Notice, from Remark 2, this means that we must be in Case 1 above: $f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) > f(S^B(\lambda_2, 1)) + f(S^A(\lambda_2, 1))$. Thus, the conditions for a split in the first period are:

1. $\bar{f}(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1, 1) - \pi^B(0)\Gamma_{\lambda_2}(0, 1) < 0$, but $\Gamma_{\lambda_1}(0, 1) + f(S^B(\lambda_1, 1)) + \pi^A(0)\Gamma_{\lambda_2}(0, 1)) < \pi^A(1)\Gamma_{\lambda_2}(1, 1)$, OR

2. $f(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1, 1) - \pi^B(0)\Gamma_{\lambda_2}(0, 1) \in (0, f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)))$, but $\Gamma_{\lambda_1}(0, 1) - \pi^B(1)\Gamma_{\lambda_2}(1, 1) + \pi^B(0)\Gamma_{\lambda_2}(0, 1) + \pi^A(0)\Gamma_{\lambda_2}(0, 1)) < \pi^A(1)\Gamma_{\lambda_2}(1, 1)$, OR

3.a $f(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1,1) - \pi^B(0)\Gamma_{\lambda_2}(0,1) > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$, but $\Gamma_{\lambda_1}(0,1) - \pi^B(1)\Gamma_{\lambda_2}(1,1) + \pi^B(0)\Gamma_{\lambda_2}(0,1) + \pi^A(0)\Gamma_{\lambda_2}(0,1)) < \pi^A(1)\Gamma_{\lambda_2}(1,1)$, OR

3.b $f(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1,1) - \pi^B(0)\Gamma_{\lambda_2}(0,1) > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$, and $\Gamma_{\lambda_1}(0,1) - \pi^B(1)\Gamma_{\lambda_2}(1,1) + \pi^B(0)\Gamma_{\lambda_2}(0,1) + \pi^A(0)\Gamma_{\lambda_2}(0,1)) > \pi^A(1)\Gamma_{\lambda_2}(1,1)$.

Setting $\pi^A = 1 - \pi^B$, Condition 2 requires $\Gamma_{\lambda_1}(0,1) + \Gamma_{\lambda_2}(0,1) - \Gamma_{\lambda_2}(1,1) < 0$, which cannot hold under the condition considered in Proposition 4: $N^A(\lambda_1, 1) + N^B(\lambda_1, 1) < N^A(\lambda_1, 0) + N^B(\lambda_1, 0)$. To see why, notice that this condition immediately implies $\Gamma_{\lambda_1}(0,1) > 0$. Furthermore, $\Gamma_{\lambda_2}(0,1) - \Gamma_{\lambda_2}(1,1) > 0$ whenever $N^A(\lambda_2, 1) + N^B(\lambda_2, 1) < N^A(\lambda_2, 0) + N^B(\lambda_2, 0)$, which is implied by $N^A(\lambda_1, 1) + N^B(\lambda_1, 1) < N^A(\lambda_1, 0) + N^B(\lambda_1, 0)$.

Next, notice that the above implies that the proposer would be willing to avoid a split even if this would cost more that the total party resources, as long as $\bar{U}^B > 0$. Hence case 3.a can never be satisfied either.

It follows that either Condition 1. holds, and the split is unilaterally induced by the proposer, or Condition 3.b holds, and the split is unilaterally induced by the receiver. A split that damages the whole camp is never consensual.

Consider next Condition 1. Setting $\pi^A = 1 - \pi^B$ and rearranging, a proposer-induced split emerges iff

$$\pi^A(1)\Gamma_{\lambda_2}(1,1) > \bar{f}(S^B(\lambda_1, 1)) + \Gamma_{\lambda_2}(1,1) - (1 - \pi^A(0))\Gamma_{\lambda_2}(0,1) \tag{34}$$

and

$$\pi^A(1)\Gamma_{\lambda_2}(1,1) > \Gamma_{\lambda_1}(0,1) + f(S^B(\lambda_1, 1)) + \pi^A(0)\Gamma_{\lambda_2}(0,1) \tag{35}$$

Recall that $\Gamma_d(1,1) < \Gamma_d(0,1)$, therefore the second condition requires $\pi^A(0) = 1 - \pi^B(0) < 1 - \pi^B(1) = \pi^A(1)$. Furthermore, notice that since the split is damaging to the camp as a whole, the first condition is never binding. Thus, a proposer-induced equilibrium emerges if and only if

$$\pi^A(1)\Gamma_{\lambda_2}(1,1) > \Gamma_{\lambda_1}(0,1) + f(S^B(\lambda_1, 1)) + \pi^A(0)\Gamma_{\lambda_2}(0,1). \tag{36}$$

This establishes a lower bound on the difference $\pi^A(1) - \pi^B(0)$. Set this difference to 1, the condition becomes

$$\Gamma_{\lambda_2}(1,1) > \Gamma_{\lambda_1}(0,1) + f(S^B(\lambda_1,1)). \tag{37}$$

The LHS is continuously increasing in λ_2, and the condition is never satisfied at $\lambda_2 = \lambda_1$ but always satisfied as $\lambda_2 \to \overline{\lambda}$. Thus, there exists a unique threshold in λ_2 s.t. the condition is satisfied iff λ_2 is above the threshold.

Next, consider Condition 3.b. A receiver-induced split emerges iff

$$f(S^B(\lambda_1,1)) + \pi^B(1)\Gamma_{\lambda_2}(1,1) - \pi^B(0)\Gamma_{\lambda_2}(0,1) > f(S^A(\lambda_1,0) + S^B(\lambda_1,0)).$$

Notice: this is the same condition as (36), up to $\pi^A = \pi^B$. This concludes the proof. □

Proof of Proposition 3. Suppose $N^A(\lambda,0)+N^B(\lambda,0) < N^A(\lambda,1)+N^B(\lambda,1)$ but $f(S^A(\lambda_2,1))+f(S^B(\lambda_2,1)) < f((S^A(\lambda_2,0) + S^B(\lambda_2,0))$. Notice, we are still in the first case outlined in the preliminaries.

Suppose that A is recognized as a proposer in the first period. A first-period split emerges if either one of these sets of conditions is satisfied:

1. $\bar{U}^B_{stable} < 0$, but $f(S^A(\lambda_1,0) + S^B(\lambda_1,0)) + \pi^A(0)\Gamma_{\lambda_2}(0,1)) < f(S^A(\lambda_1,1)) + \pi^A(1)\Gamma_{\lambda_2}(1,1)$, OR

2. $\bar{U}^B_{stable} \in (0, f(S^A(\lambda_1,0)+S^B(\lambda_1,0)))$, but $f(S^A(\lambda_1,0)+S^B(\lambda_1,0))-\bar{U}^B_{stable}+\pi^A(0)\Gamma_{\lambda_2}(0,1)) < f(S^A(\lambda_1,1)) + \pi^A(1)\Gamma_{\lambda_2}(1,1)$, OR

3. $\bar{U}^B_{stable} > f(S^A(\lambda_1,0) + S^B(\lambda_1,0))$.

First, consider the set of conditions 1 (split is unilaterally induced by faction A). These conditions can be rewritten as:

$$(1-\pi^A(0))\Gamma_{\lambda_2}(0,1) > \bar{f}(S^B(\lambda_1,1)) + (1-\pi^A(1))\Gamma_{\lambda_2}(1,1) \tag{38}$$

and

$$\pi^A(1)\Gamma_{\lambda_2}(1,1) > \Gamma_{\lambda_1}(0,1) + f(S^B(\lambda_1,1)) + \pi^A(0)\Gamma_{\lambda_2}(0,1) \tag{39}$$

Recall that $\Gamma_{\lambda_2}(0,1) < \Gamma_{\lambda_2}(1,1)$, therefore the first condition requires $\pi^A(1) > \pi^A(0)$. Under this assumption, both conditions establish a lower bound on λ_2. Notice that this lower bound may be smaller or larger than λ_1, depending on the effect of the split and the curvature of f.

Next, consider the second case: the split is conflictual and initiated by faction A, since $\bar{U}^B_{stable} \in \big(0, f(S^A(\lambda_1,0) + S^B(\lambda_1,0))\big)$, but the proposer A prefers not to extend such an offer. Such split emerges in equilibrium if and only if:

$$\Gamma_{\lambda_1}(0,1) < \Gamma_{\lambda_2}(1,1) - \Gamma_{\lambda_2}(0,1), \tag{40}$$

Notice that the RHS of this condition is increasing in λ_2, since the split increases the camp's support, and the condition is always satisfied as $\lambda_2 \to \overline{\lambda}$. Thus, there exists a unique $\hat{\lambda}^p$ s.t. the proposer wants a split iff $\lambda_2 > \hat{\lambda}^p$. Notice that this bound may be lower or higher than λ_1, depending on the effect of the split and the shape of f.

Finally, consider the third case, $\bar{U}^B_{stable} > f(S^A(\lambda_1,0) + S^B(\lambda_1,0))$. A split emerges if an only if:[20]

$$\pi^B(1)\Gamma_{\lambda_2}(1,1) - \pi^B(0)\Gamma_{\lambda_2}(0,1) - \Gamma_{\lambda_1}(0,1) - f(S^A(\lambda_1,1)) > 0, \tag{41}$$

which we can rearrange as:

$$\Gamma_{\lambda_1}(0,1) < \pi^B(1)\Gamma_{\lambda_2}(1,1) - \pi^B(0)\Gamma_{\lambda_2}(0,1) - f(S^A(\lambda_1,1)). \tag{42}$$

Differentiating the RHS wrt λ_2 we obtain that it is increasing iff

$$\pi^B(1)\frac{\partial\Gamma_{\lambda_1}(1,1)}{\partial\lambda_2} - \pi^B(0)\frac{\partial\Gamma_{\lambda_1}(0,1)}{\partial\lambda_2} \tag{43}$$

[20]Notice that such a split is consensual if condition (40) holds, conflictual otherwise.

Recall that $\frac{\partial \Gamma_{\lambda_1}(1,1)}{\partial \lambda_2} > 0$ given convexity, thus the LHS is increasing in $\pi^B(1)$. Furthermore, the condition is always satisfied at $\pi^B(1) = \pi^B(0)$ (because $\frac{\partial \Gamma_{\lambda_1}(1,1)}{\partial \lambda_2} - \frac{\partial \Gamma_{\lambda_1}(0,1)}{\partial \lambda_2} > 0$ when the split increases support) and never at $\pi^B(1) = 0$. Thus, there exists a unique $\hat{\pi}^B(1) < \pi^B(0)$ s.t. the LHS of 42 is increasing in λ_2 iff $\pi^B(1) > \hat{\pi}^B(1)$. Thus, suppose first that $\pi^B(1) > \hat{\pi}^B(1)$. Furthermore, notice that (42) is never satisfied as $\lambda_2 \to \underline{\lambda}$ and always satisfied as $\lambda_2 \to \bar{\lambda}$. Thus, there exists a unique $\hat{\lambda}^r_{ben}$ s.t. (42) is satisfied iff $\pi^B(1) > \hat{\pi}^B(1)$ and $\lambda_2 > \hat{\lambda}^r_{ben}$.

Combining the three cases, there exists a unique cutoff in λ_2 s.t. a split emerges in the first period iff λ_2 is above this cutoff. The binding cutoff is defined either by condition 39, 40 or 42, depending on parameter values.[21] □

Proof of Proposition 4. Suppose $f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) < f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1))$: the case that the split helps the camp as a whole, *and* the effect is so large that staying together eliminates the surplus in the second period. This implies that if a merger remains stable in the first period, this simply delays the split until the second.

In this case, intuitively, a split on the equilibrium path may never be avoided. Thus, the equilibrium takes one of two forms: split-merge, or merge-split.

A first-period split emerges if either one of these sets of conditions is satisfied

1. $f(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1, 1) < f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$, but $f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) - \Big(f(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1, 1)\Big) < f(S^A(\lambda_1, 1)) + \pi^A(1)\Gamma_{\lambda_2}(1, 1)$, OR
2. $\bar{f}(S^B(\lambda_1, 1)) + \pi^B(1)\Gamma_{\lambda_2}(1, 1) > f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0))$.

Consider the first case. Substituting the value of $\bar{U}^B$ into A's problem and setting $\pi^A = 1 - \pi^B$, the proposer wants to trigger a split iff

$$\Gamma_{\lambda_1}(0, 1) < \Gamma_{\lambda_2}(1, 1) \tag{44}$$

First, notice that the RHS is continuously increasing in λ_2. Furthermore, since $f(S^A(\lambda_2, 0) + S^B(\lambda_2, 0)) < f(S^A(\lambda_2, 1)) + f(S^B(\lambda_2, 1))$, the above is always satisfied when $\lambda_1 = \lambda_2$. Thus,

[21] Notice that the cutoff always exists, regardless of whether $\pi^B(1)$ is larger or smaller than $\pi^B(0)$, because cases 1 and 3 cover both possibilities.

there exists a unique $\tilde{\lambda}^p < \lambda_1$ s.t., the proposer triggers a split iff $\lambda_2 > \tilde{\lambda}^p$. This is because, with a split that increases the camp's strength, this effect is amplified by a higher λ_2, and therefore a higher λ_2 incentivizes the proposer to even pay the immediate cost.

Finally, consider the second case. The receiver wants to trigger a split iff

$$\Gamma_{\lambda_1}(0,1) < \pi^B(1)\Gamma_{\lambda_2}(1,1) - f(S^A(\lambda_1,1)) \tag{45}$$

Notice that the RHS is increasing in λ_2 and the condition is always satisfied when $\lambda_t \to \overline{\lambda}$. Thus, there exists a lower bound $\tilde{\lambda}$ s.t. the condition holds iff $\lambda_2 > \tilde{\lambda}$. Furthermore, the RHS of 44 is higher than the RHS of 45, but the LHS is the same. Thus, $\tilde{\lambda}^p < \tilde{\lambda}$. □

Proof of Proposition 6. When the split reduces the total support of the camp, by Proposition 2, the split is never consensual: it is either unilaterally initiated by the proposer or by the receiver. We will consider a receiver-initiated split (the proof for the proposer is analogous).

Suppose A is recognized in period 1. A receiver-initiated split in period 1 occurs iff

$$\begin{aligned} &f(S^B(\lambda_1,1)) + \pi^B(1)\big(f(S^A(\lambda_2,1)+S^B(\lambda_2,1)) - f(S^A(\lambda_2,1)) - f(S^B(\lambda_2,1))\big) > \\ &f(S^A(\lambda_1,0)+S^B(\lambda_1,0)) + \pi^B(0)\big(f(S^A(\lambda_2,0)+S^B(\lambda_2,0)) - f(S^A(\lambda_2,1)) - f(S^B(\lambda_2,1))\big) \end{aligned} \tag{46}$$

that is

$$f(S^B(\lambda_1,1)) - f(S^A(\lambda_1,0)+S^B(\lambda_1,0)) + \pi^B(1)\Gamma_{\lambda_2}(1,1) - \pi^B(0)\Gamma_{\lambda_2}(0,1) > 0.$$

Suppose $\pi^B(1) \leq \pi^B(0)$: in this case, condition (46) is never satisfied by assumption of damaging split. Suppose $\pi(1) = 1$ and $\pi(0) = 0$. In this case, condition (46) reduces to

$$f(S^B(\lambda_1,1)) + \Gamma_{\lambda_2}(1,1) - f(S^A(\lambda_1,0)+S^B(\lambda_1,0)) > 0, \tag{47}$$

which is positive for $\lambda_2 > \bar{\lambda}$.

Substituting $\pi^B(\rho^B) = \frac{1}{2} + \phi\rho^B$ into (46), we obtain:

$$f(S^B(\lambda_1, 1)) - f(S^A(\lambda_1, 0) + S^B(\lambda_1, 0)) + \phi\left[\rho^B(1)\Gamma_{\lambda_2}(1, 1) - \rho^B(0)\Gamma_{\lambda_2}(0, 1)\right] > 0.$$

Notice that, if $\rho^B(1)\Gamma_{\lambda_2}(1, 1) - \rho^B(0)\Gamma_{\lambda_2}(0, 1) < 0$, the condition can never be satisfied since the split reduces the camp's total support. Thus, necessary condition for a split to emerge in equilibrium is that $\rho^B(1)\Gamma_{\lambda_2}(1, 1) - \rho^B(0)\Gamma_{\lambda_2}(0, 1) > 0$. Under this condition, a decrease in ϕ (a more egalitarian party organization) decreases the likelihood of splitting. Thus, decreasing ϕ either has no impact on the likelihood of a split (in the sense of set inclusion), or strictly decreases it. □

Proof of Proposition 7. Consider the case where a split that, while statically inefficient in both periods, nonetheless increases the camp's total support, as in Proposition 3. In this case, a receiver-induced split emerges whenever the continuation value of the receiver exceeds the total party resources in period 1. Suppose without loss of generality that B is the receiver in period 1. This condition can be written as

$$\Gamma_{\lambda_1}(0, 1) < \pi^B(1)\Gamma_{\lambda_2}(1, 1) - \pi^B(0)\Gamma_{\lambda_2}(0, 1) - f(S^A(\lambda_1, 1)). \tag{48}$$

Because $\Gamma_{\lambda_2}(1, 1) > \Gamma_{\lambda_2}(0, 1)$ when a split increases the camp's total support, the inequality can be satisfied when $\pi^B(1) < \pi^B(0)$ and thus $\Delta^B < 0$. In this case, making the party more egalitarian, i.e., increasing Δ^B actually makes splits *more* likely.

Under some parameter values, this condition is the binding one for a split to emerge in equilibrium. In particular, from the proof of Proposition 5 we can see that this is true when two conditions are satisfied. First, $\pi^B(1) < 1/2 < \pi^B(0)$, and (consistently) $S^A(\lambda, 1) > S^B(\lambda, 1)$, which ensures that 42 is more binding than 39. Second, the effect of the split on total support is sufficiently small, so that $\Gamma_d(0, 1)$ is close to $\Gamma_d(1, 1)$, which ensures that 42 is more binding than 40. □